Appendix C

Phase Space Gallery II: Amplitude-Modulated Pendulum

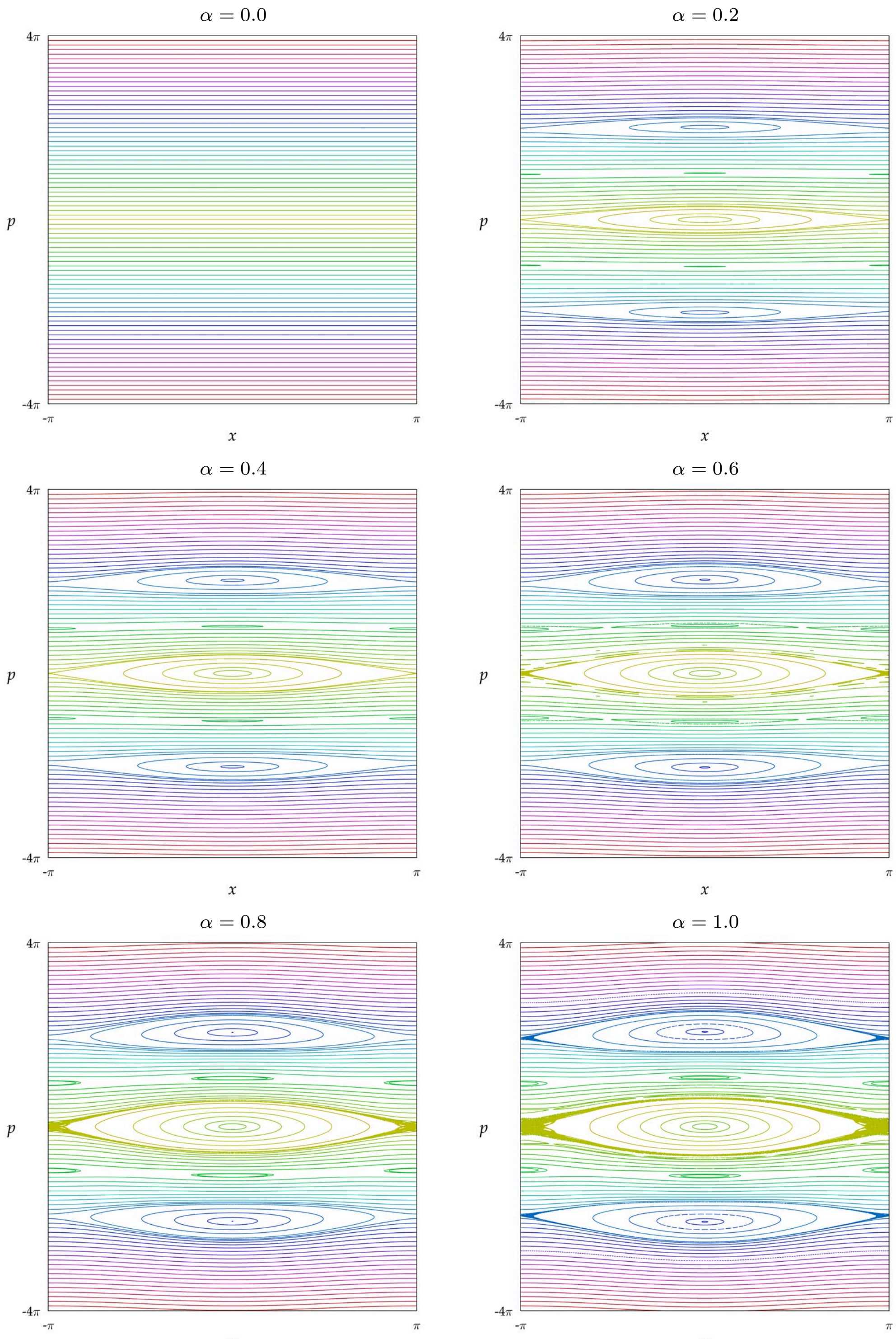
Now we examine the phase space for the other system that we study in this dissertation, the amplitude-modulated pendulum. This system is described by the Hamiltonian

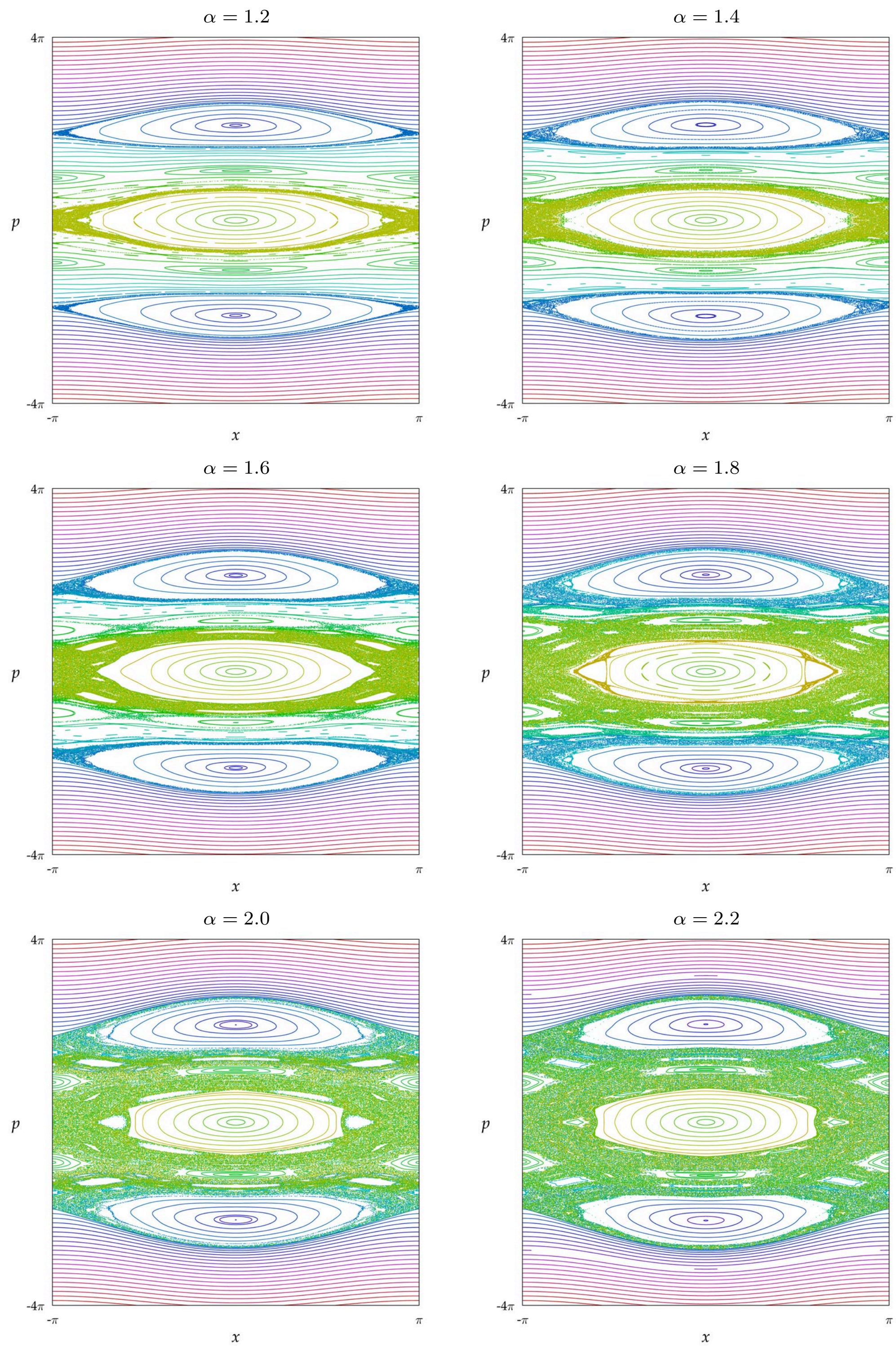
$$H(x,p,t) = \frac{p^2}{2} - 2\alpha\cos^2(\pi t)\cos(x) \quad , \tag{C.1}$$

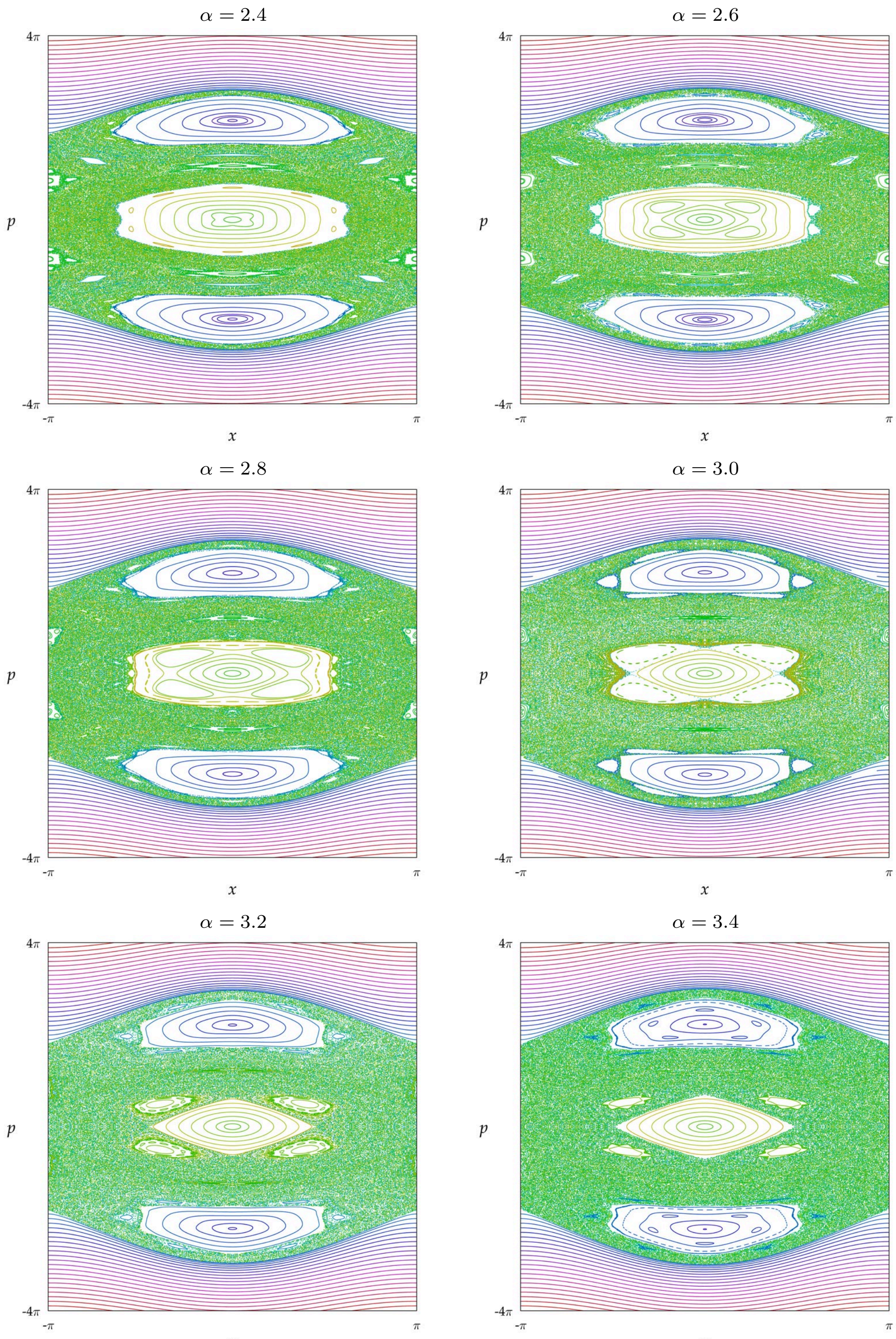
which is just the pendulum Hamiltonian with a single-frequency modulation of the well depth. As the time dependence here can be decomposed into three frequencies, the phase-space is dominated by the three corresponding primary resonances, located at $p = 0$ and $p = \pm 2\pi$. These three resonances form as α increases from zero, and they dissolve into the surrounding chaotic sea as α continues to grow. Especially dramatic is the "molting" behavior of the islands, where they grow an island chain and then shed it into the chaotic sea; this can be seen, for example, for a period-4 chain in the center island around $\alpha = 3.2$, and a period-4 island chain in the outer islands around $\alpha = 5.2$. Also interesting is that the remnants of the center island disappear around $\alpha = 11$, but the island makes a strong reappearance around $\alpha = 18$.

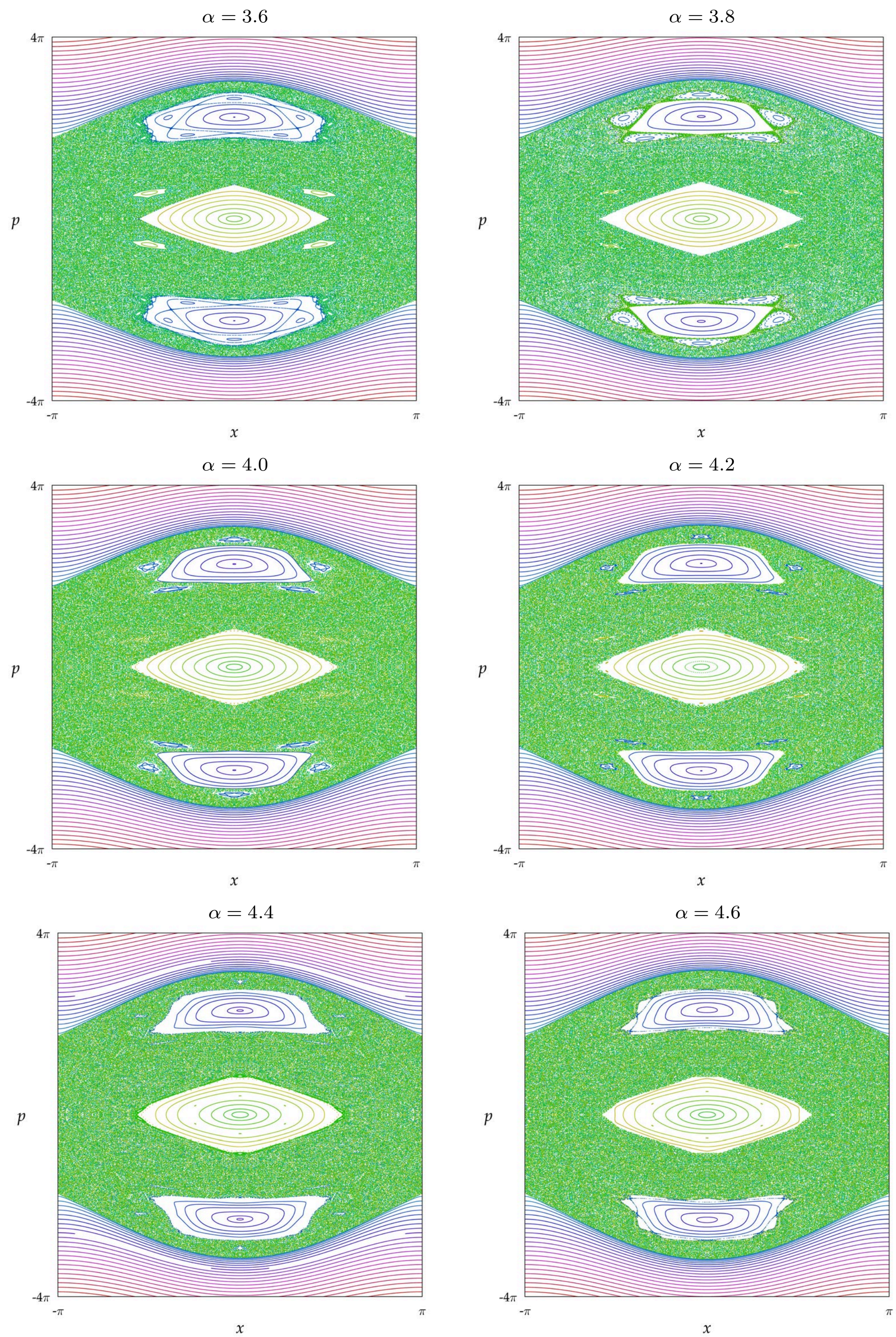
In the following graphics, the trajectory coordinates are plotted, sampled at unit times $t = n$ (for integer n). The phase plots here show about 60 different trajectories, with around 4000 iterations per trajectory. To retain the symmetry of the phase space, the three symmetric images $(x, -p)$, $(-x, p)$, and $(-x, -p)$ of each (x, p) point is also plotted.

The phase-space plots in this gallery were again hand-coded directly in POSTSCRIPT, where the code contained two embedded integrators, a fixed-step, second-order Stoermer routine and a fourth-order Runge-Kutta routine. Despite the lower order of the Stoermer method, it was much more accurate for the same step size than the Runge-Kutta integrator. These graphics files were again rasterized before inclusion in the PDF-formatted version of this document, as they require extensive processing time compared to the standard-map phase plots.

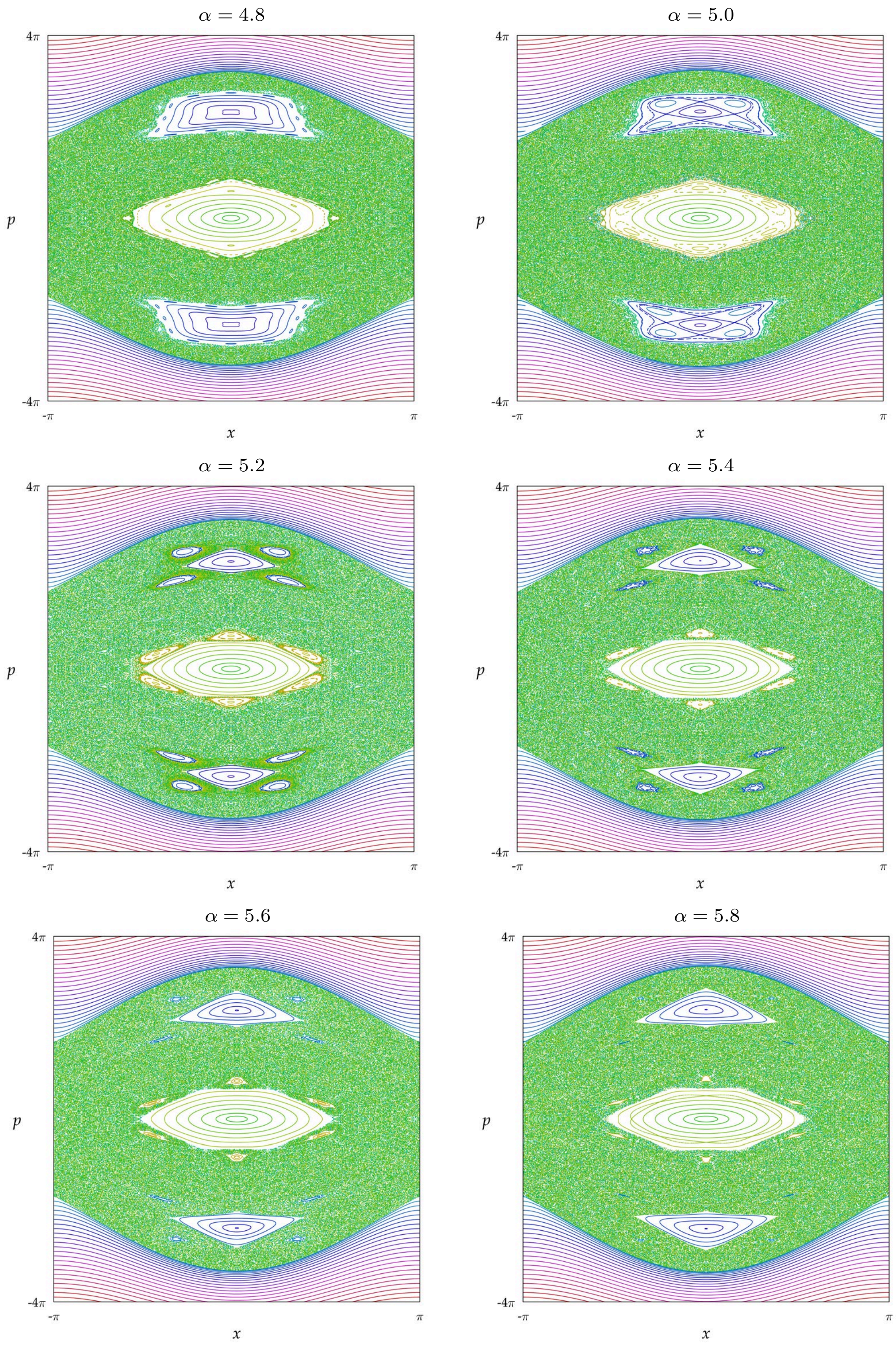

α = 0.0
α = 0.2
α = 0.4
α = 0.6
α = 0.8
α = 1.0
4π
-4π
p
-π
π
x

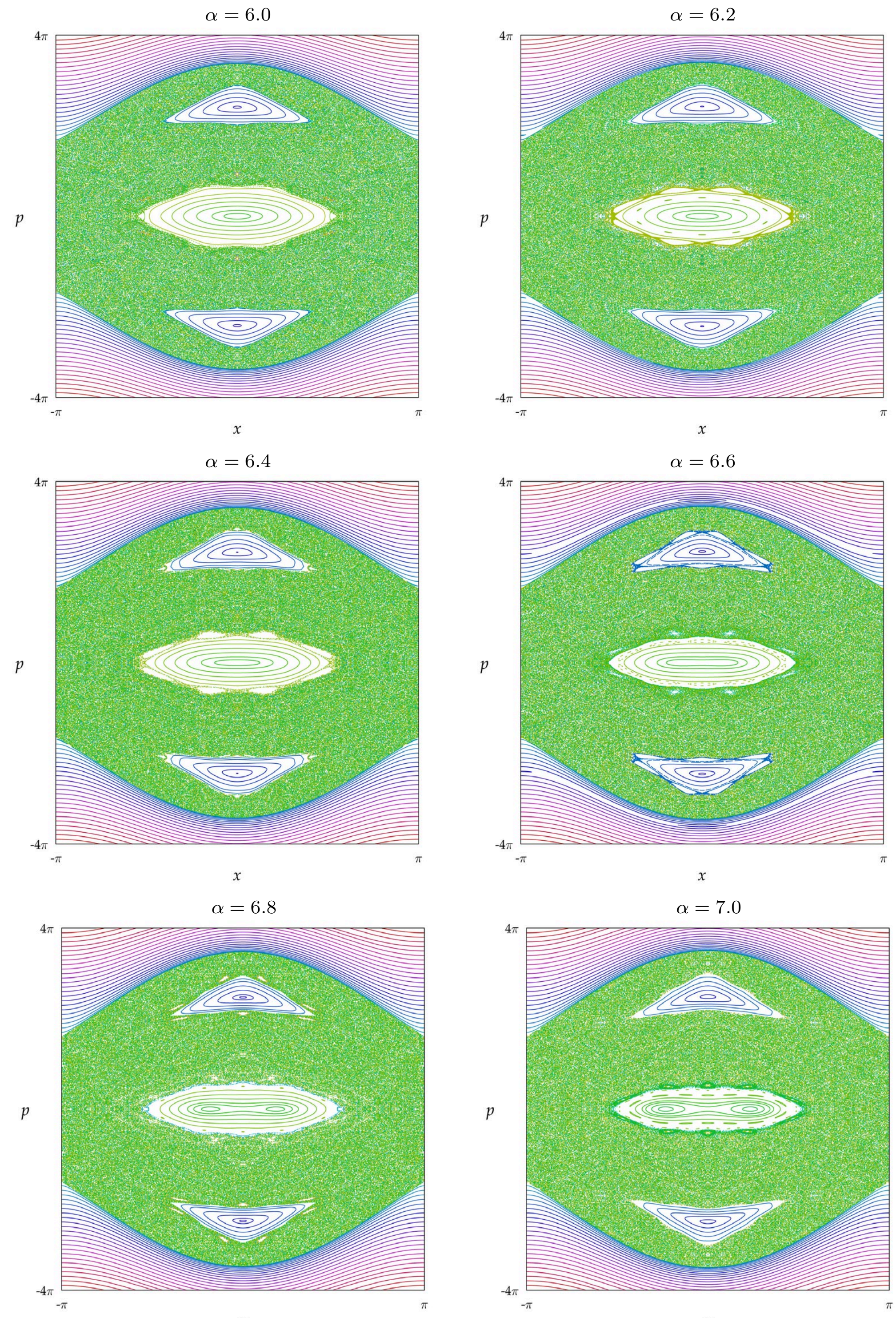

$\alpha = 1.2$
$\alpha = 1.4$
$\alpha = 1.6$
$\alpha = 1.8$
$\alpha = 2.0$
$\alpha = 2.2$
4π
-4π
$-\pi$
π
p
x


$\alpha = 2.4$
$\alpha = 2.6$
$\alpha = 2.8$
$\alpha = 3.0$
$\alpha = 3.2$
$\alpha = 3.4$
4π
-4π
$-\pi$
π
p
x



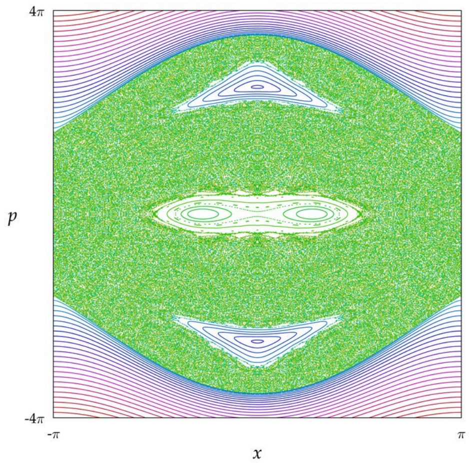
$\alpha = 3.6$
$\alpha = 3.8$
$\alpha = 4.0$
$\alpha = 4.2$
$\alpha = 4.4$
$\alpha = 4.6$
4π
-4π
$-\pi$
π
p
x


$\alpha = 4.8$
$\alpha = 5.0$
$\alpha = 5.2$
$\alpha = 5.4$
$\alpha = 5.6$
$\alpha = 5.8$
4π
-4π
p
$-\pi$
π
x

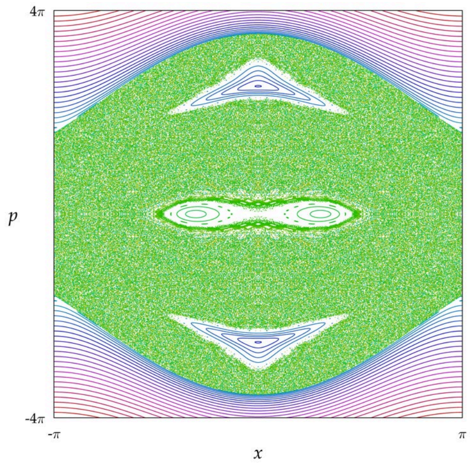


α = 6.0
α = 6.2
α = 6.4
α = 6.6
α = 6.8
α = 7.0
4π
-4π
-π
π
p
x

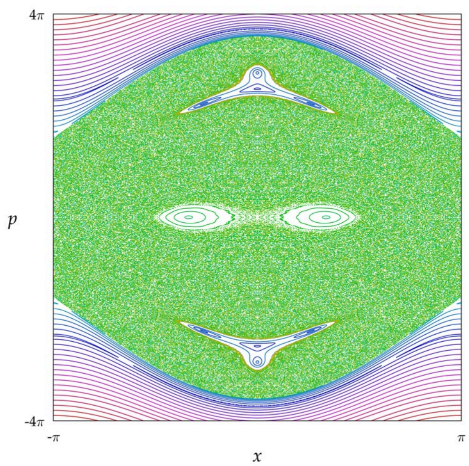
$\alpha = 7.2$



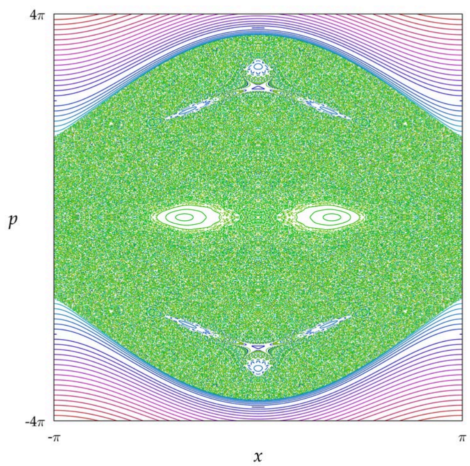
$\alpha = 7.4$



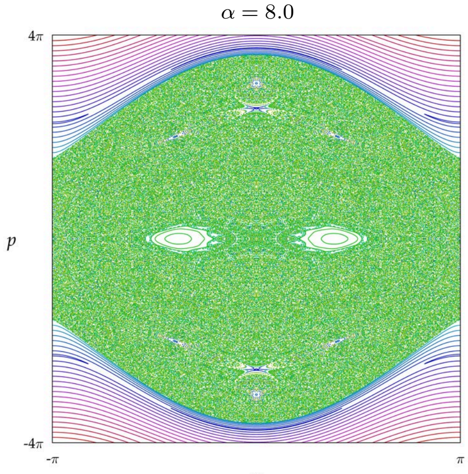
$\alpha = 7.6$



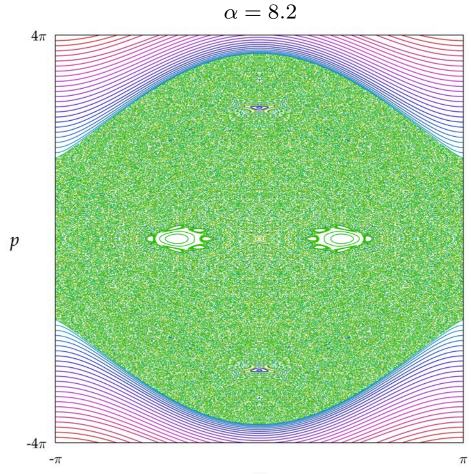
$\alpha = 7.8$

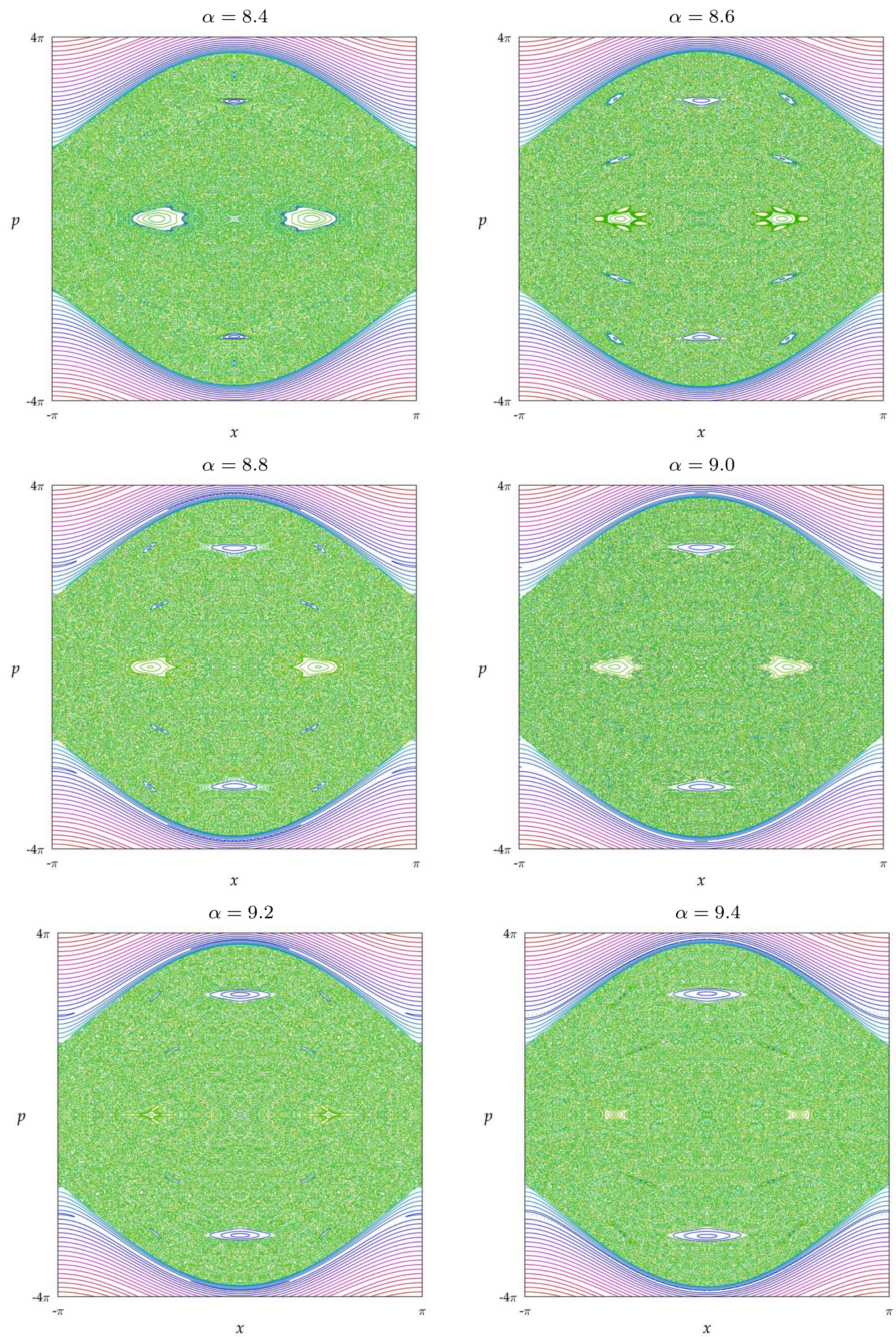


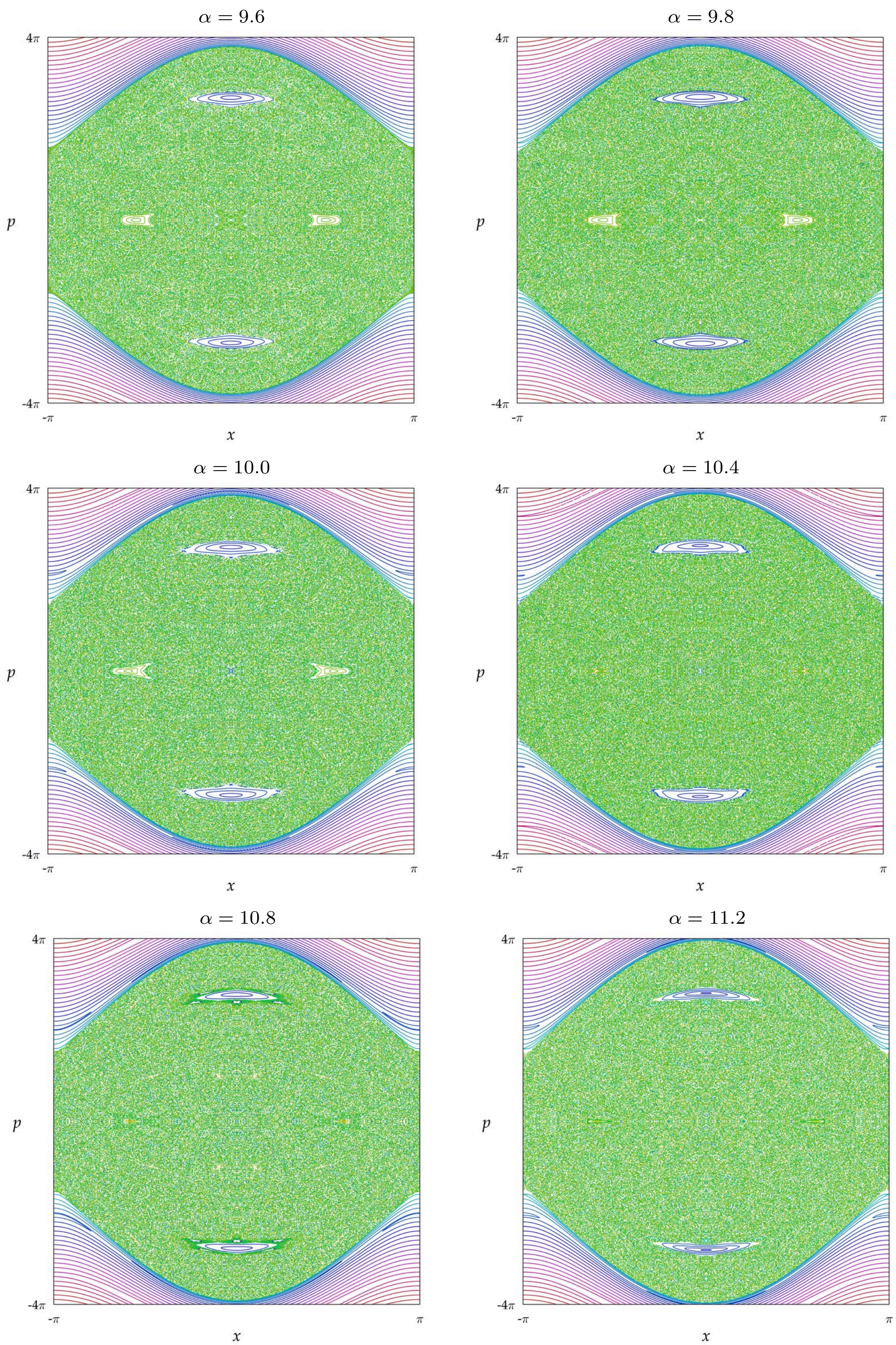
$\alpha = 8.0$

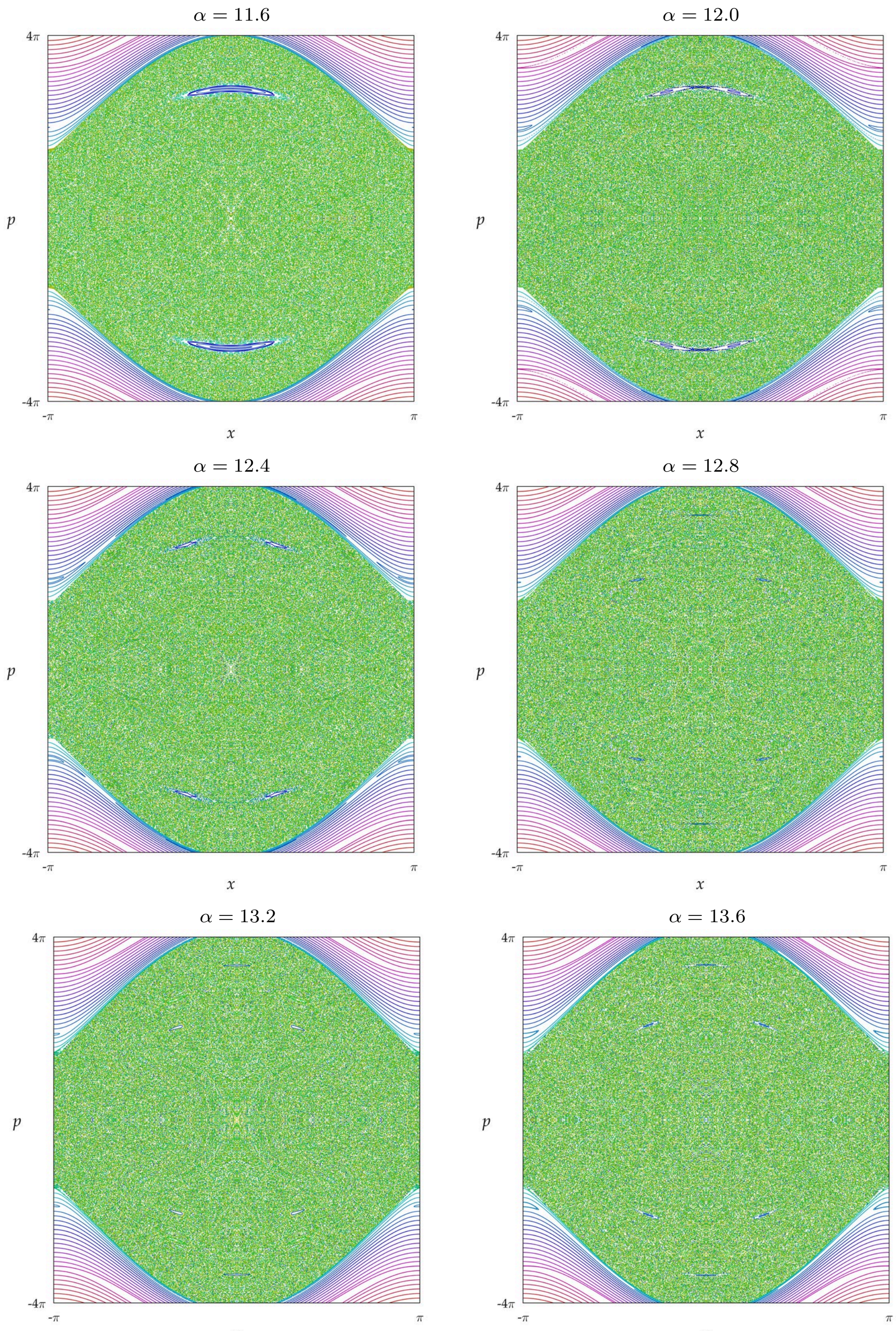


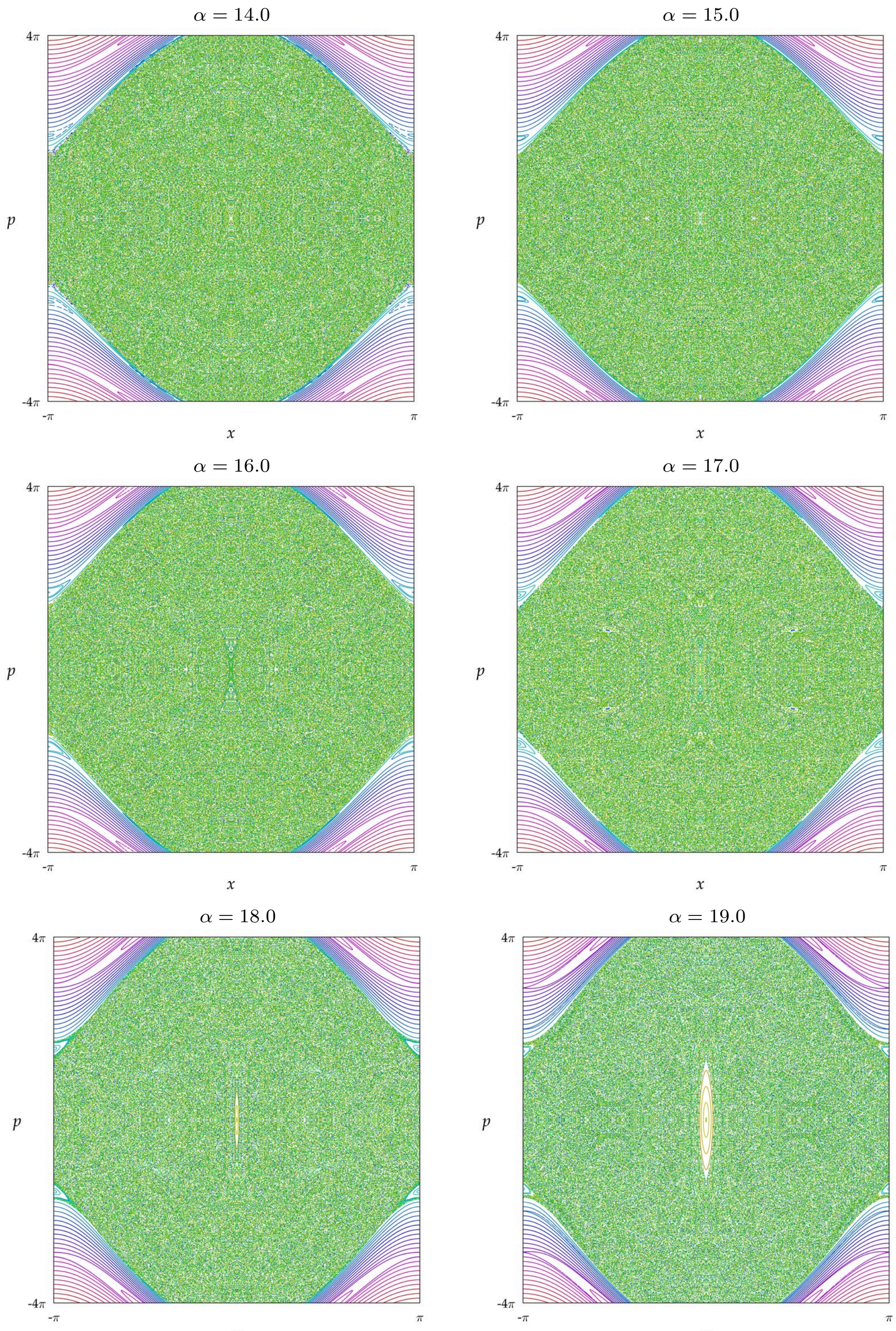
$\alpha = 8.2$

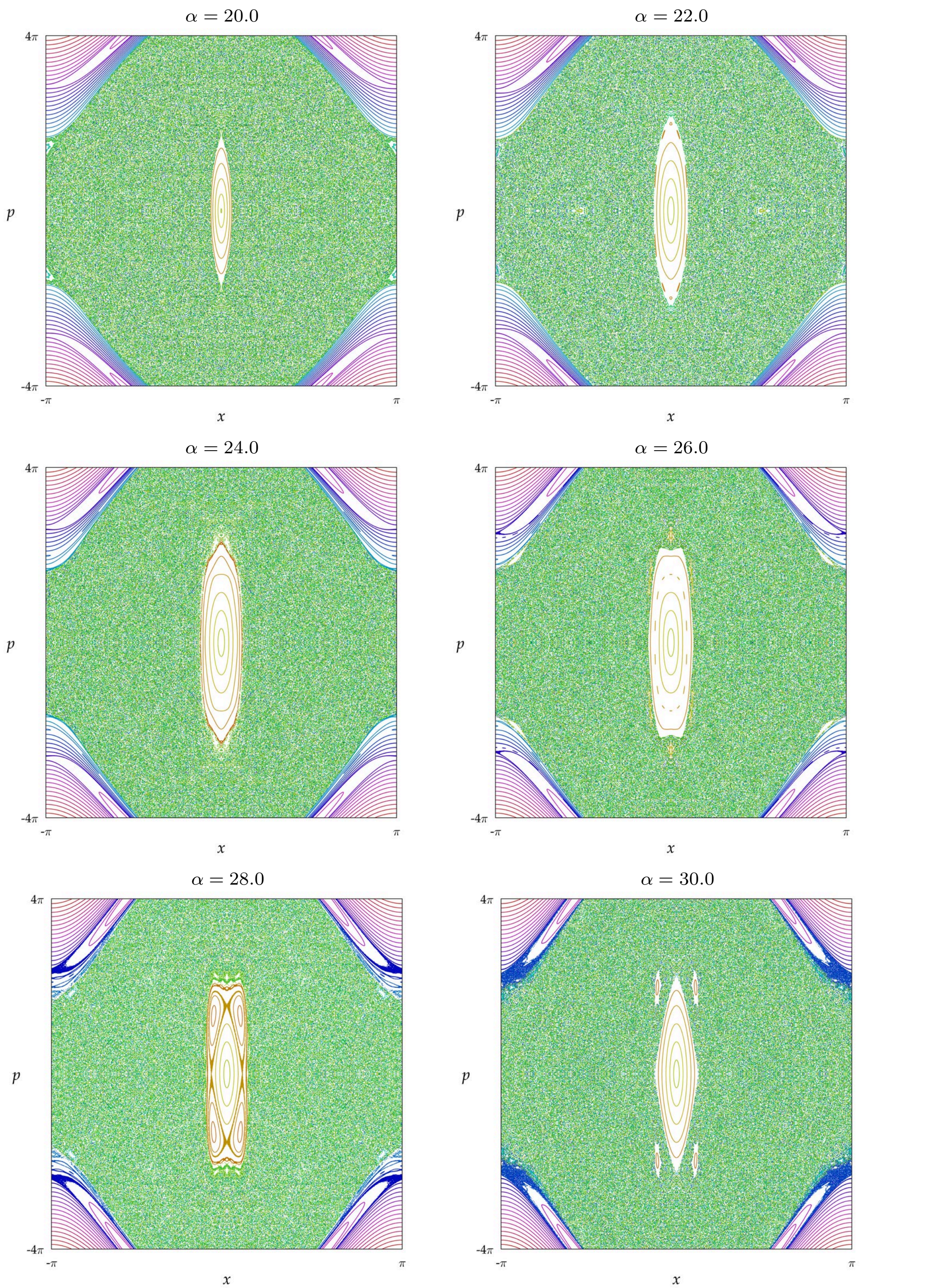



$\alpha = 8.4$
$\alpha = 8.6$
$\alpha = 8.8$
$\alpha = 9.0$
$\alpha = 9.2$
$\alpha = 9.4$
4π
-4π
$-\pi$
π
p
x


$\alpha = 9.6$
$\alpha = 9.8$
$\alpha = 10.0$
$\alpha = 10.4$
$\alpha = 10.8$
$\alpha = 11.2$
4π
-4π
$-\pi$
π
p
x


α = 11.6
α = 12.0
α = 12.4
α = 12.8
α = 13.2
α = 13.6
4π
-4π
-π
π
p
x


$\alpha = 14.0$
$\alpha = 15.0$
$\alpha = 16.0$
$\alpha = 17.0$
$\alpha = 18.0$
$\alpha = 19.0$
4π
-4π
p
$-\pi$
π
x



$\alpha = 20.0$
$\alpha = 22.0$
$\alpha = 24.0$
$\alpha = 26.0$
$\alpha = 28.0$
$\alpha = 30.0$
4π
-4π
$-\pi$
π
p
x