Quantum Optics Project I

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1 Problem I

The Bloch equations for the two-level atom were integrated using numerical methods. The following are the optical Bloch equations when homogenous broadening is assumed:

$$\begin{aligned}
\partial_t\langle\sigma_x\rangle &= \Delta\langle\sigma_y\rangle - \tfrac{\Gamma}{2}\langle\sigma_x\rangle \\
\partial_t\langle\sigma_y\rangle &= -\Delta\langle\sigma_x\rangle - \Omega\langle\sigma_z\rangle - \tfrac{\Gamma}{2}\langle\sigma_y\rangle \\
\partial_t\langle\sigma_z\rangle &= \Omega\langle\sigma_y\rangle - \Gamma(\langle\sigma_z\rangle + 1)
\end{aligned} \tag{1}$$

1.1 Numerical Solution

The numerical solution is expected to demonstrate damped Rabi oscillations between the two energy levels and maintain the normalization $\rho_{ee} + \rho gg = 1$ for the populations of the two energy levels. Choosing arbitrary parameters, damped Rabi oscillations are observed as Figure 1 depicts.

The normalization was tested and found to be consistent for all times. Also, for zero detuning we expect $\langle\sigma_x\rangle = 0$ for all times when $\langle\sigma_x(0)\rangle = 0$, which was seen in the output data.

1.2 Analytical Comparison

The numerical solution was compared to two analytical solutions to test its accuracy. The steady state solution is given by:

$$\rho_{ee}(\rightarrow \text{inf}) = \frac{\frac{\Omega^2}{\Gamma^2}}{1 + \frac{2*\delta}{\Gamma}^2 + 2 * \frac{\Omega^2}{\Gamma^2}} \tag{2}$$

$$\rho_{eg}(\rightarrow \text{inf}) = -i\frac{\Omega}{\Gamma}\frac{1 + \frac{i2\Delta}{\Gamma}}{1 + (\frac{2\Delta}{\Gamma})^2 + 2\frac{\Omega^2}{\Gamma^2}} \tag{3}$$

Also, for zero detuning and weak excitation, the Torrey solution is given by:

$$\langle\sigma_x(t)\rangle = 0$$
$$\langle\sigma_y(t)\rangle = \frac{\Omega\Gamma}{\Omega^2+\frac{\Gamma^2}{2}}\left[1-e^{-\frac{3\Gamma}{4}t}\left(cos\Omega_\Gamma t - \frac{\Omega^2-\frac{\Gamma^2}{4}}{\Gamma\Omega_\Gamma}sin\Omega_\Gamma t\right)\right] \tag{4}$$
$$\langle_z(t)\rangle = -1+\frac{\Omega^2}{\Omega^2+\frac{\Gamma^2}{2}}\left[1-e^{-\frac{3\Gamma}{4}t}\left(cos\Omega_\Gamma t + \frac{3\Gamma}{4\Omega_\Gamma}sin\Omega_\Gamma t\right)\right]$$

where

$$\Omega_\Gamma = \sqrt{\Omega^2 - \left(\frac{\Gamma}{4}\right)^2} \tag{5}$$

The following graphs depict ρ_{ee} vs. time and $\langle\sigma_y\rangle$ vs. time for the numerical solution as well as for the two analytical ones:

The numerical solution is nearly indistinguishable from the Torrey solution and approaches the predicted steady state value. As shown in Figures 2 and 3.

2 Problem II

2.1 Introduction

In this problem we are to compare solutions of the unconditioned master equation to the stochastic Schrödinger equation for a two-level system. It will be shown that when a large number of trajectories are averaged over for the stochastic process the resulting populations mirror that of a damped system.

Beginning with

$$\partial_t\rho = \frac{1}{i\hbar}[H_\mathrm{A}+H_\mathrm{AF},\rho] + \Gamma D[\sigma]\rho = \frac{1}{i\hbar}[H_\mathrm{A}+H_\mathrm{AF},\rho] - \frac{\Gamma}{2}[\sigma^\dagger\sigma,\rho]_+ + \Gamma\sigma\rho\sigma^\dagger, \tag{6}$$

, if we modify it to include measurement, the master equation becomes

$$d\rho = \frac{1}{i\hbar}[H_\mathrm{A}+H_\mathrm{AF},\rho]dt - \frac{\Gamma}{2}[\sigma^\dagger\sigma,\rho]_+dt + \Gamma\langle\sigma^\dagger\sigma\rangle\rho dt + (\frac{\sigma\rho\sigma^\dagger}{\langle\sigma^\dagger\sigma\rangle} - \rho)dN. \tag{7}$$

Where dN acts to put the system back into the ground state. We model this as the detection of a photon or the "Steckism click".

Naturally, when modelling a two level system it is advantageous to solve for the wave function rather than the density matrix. Therefore we would like to use the stochastic Schrödinger equation, which is less intense computationally. It can be shown that expanding $d\rho$ to second order in dt and plugging in

$$d|\psi\rangle = \frac{1}{i\hbar}(H_\mathrm{A}+H_\mathrm{AF})|\psi\rangle dt + \frac{\Gamma}{2}(\langle\sigma^\dagger\sigma\rangle - \sigma^\dagger\sigma)|\psi\rangle dt + (\frac{\sigma}{\sqrt{\langle\sigma^\dagger\sigma\rangle}} - 1)|\psi\rangle dN. \tag{8}$$

results in the stochastic master equation.

The above equation, with $dN = 0$, models the evolution of a single atom without being detected. When a detection takes place the last term returns the atom to the ground state.

The probability of an atom emitting a photon is given by

$$\Delta N = \int_t^{t+\Delta t} dN = \Gamma\overline{\langle\sigma^\dagger\sigma\rangle}\Delta t, \tag{9}$$

where Δt is the minimum time between detections (a parameter of the detector used) and

$$\overline{\langle\sigma^\dagger\sigma\rangle} \approx \frac{\langle\sigma^\dagger\sigma\rangle(t) + \langle\sigma^\dagger\sigma\rangle(t+\Delta t)}{2} \tag{10}$$

is the average excited-state population during the interval $[t, t+\Delta t)$.

2.2 Results

We would like to show a progression in averaging trajectories, from 1, 5, 20, 1000, and 20,000 (Figures 4 through 8) in comparison to the unconditioned solution (Figure 9). With one trajectory you can clearly see detector clicks represented by the quantum jumps back to the ground state. There are approximately seven jumps visible in Figure 4. As the number of trajectories averaged increases the signal becomes less noisy and when several thousand trajectories are averaged the signal converges to that of the unconditioned solution.

3 Problem III

3.1 Introduction

In modelling a vee atom we will use the same stochastic Schrödinger equation as used in problem II,

$$d|\psi\rangle = \frac{1}{i\hbar}(H_{\mathrm{A}} + H_{\mathrm{AF}})|\psi\rangle dt + \frac{\Gamma}{2}(\langle\sigma^\dagger\sigma\rangle - \sigma^\dagger\sigma)|\psi\rangle dt + (\frac{\sigma}{\sqrt{\langle\sigma^\dagger\sigma\rangle}} - 1)|\psi\rangle dN. \tag{11}$$

However, we will be using a modified atom-field Hamiltonian to account for the second excited state,

$$\begin{aligned} H_{\mathrm{A}} &= -\hbar\Delta_1|e_1\rangle\langle e_1| - \hbar\Delta_2|e_2\rangle\langle e_2|, \\ H_{\mathrm{AF}} &= \frac{\hbar\Omega_1}{2}(\sigma_1 + \sigma_1^\dagger) + \frac{\hbar\Omega_2}{2}(\sigma_2 + \sigma_2^\dagger), \end{aligned} \tag{12}$$

with everything in the rotating frame. Also, in order to treat the detector clicks we assume that we are unable to distinguish which channel the atom takes to the ground state. To do this we simply take,

$$\sqrt{\Gamma}\sigma \rightarrow \sqrt{\Gamma_1}\sigma_1 + \sqrt{\Gamma_2}\sigma_2 \tag{13}$$

$$\begin{aligned} \Gamma\sigma^\dagger\sigma &\rightarrow \Gamma_1|c_{e_1}\rangle\langle c_{e_1}| + \Gamma_2|c_{e_2}\rangle\langle c_{e_2}| + \sqrt{\Gamma_1\Gamma_2}(|c_{e_1}\rangle\langle c_{e_2}| + |c_{e_2}\rangle\langle c_{e_1}|), \\ \Gamma\langle\sigma^\dagger\sigma\rangle &\rightarrow \Gamma\rho_{e_1} + \Gamma\rho_{e_2} + \sqrt{\Gamma_1\Gamma_2}(c_{e_1}c_{e_2}^* + c_{e_1}^* c_{e_2}). \end{aligned} \tag{14}$$

Then the stochastic Schrödinger equation becomes

$$\begin{aligned} \dot{c_{e_1}} &= (i\Delta_1 - \frac{\Gamma}{2}\langle\sigma^\dagger\sigma\rangle - \frac{\Gamma_1}{2})c_{e_1} - \frac{\sqrt{\Gamma_1\Gamma_2}}{2}c_{e_2} - \frac{i\Omega_1}{2}c_g, \\ \dot{c_{e_2}} &= -\frac{\sqrt{\Gamma_1\Gamma_2}}{2}c_{e_1} + (i\Delta_2 - \frac{\Gamma}{2}\langle\sigma^\dagger\sigma\rangle - \frac{\Gamma_2}{2})c_{e_2} - \frac{i\Omega_2}{2}c_g, \\ \dot{c_g} &= -\frac{i\Omega_1}{2}c_{e_1} - \frac{i\Omega}{2}c_{e_2} + \frac{\sqrt{\Gamma_1\Gamma_2}}{2}c_g. \end{aligned} \tag{15}$$

3.2 Results

In Figure 10 we see population of atoms oscillate between the two excited states given the same oscillator strength, and same decay rate. Also, the ensemble was initially in the same excited state.

In Figures 11 and 12, quantum jumps are visible. This is seen as the randomly blinking on and off of transitions depending on the state of the atom. The fast oscillations represent fluorescence of the fast decay and hard driven transition. At some points the atom got stuck in the slow decay and weakly driven excited state for an extended amount of time. When the atom is stuck in the slow decay channel the atom cannot fluoresce. This is seen as the atom being stuck in a darkstate. Unfortunately, we found a consistent divergence (Figure 11) in our calculations. If the atom stays in an excited for too long the population blows up, this is not physical. We believe this may be do to round off error.

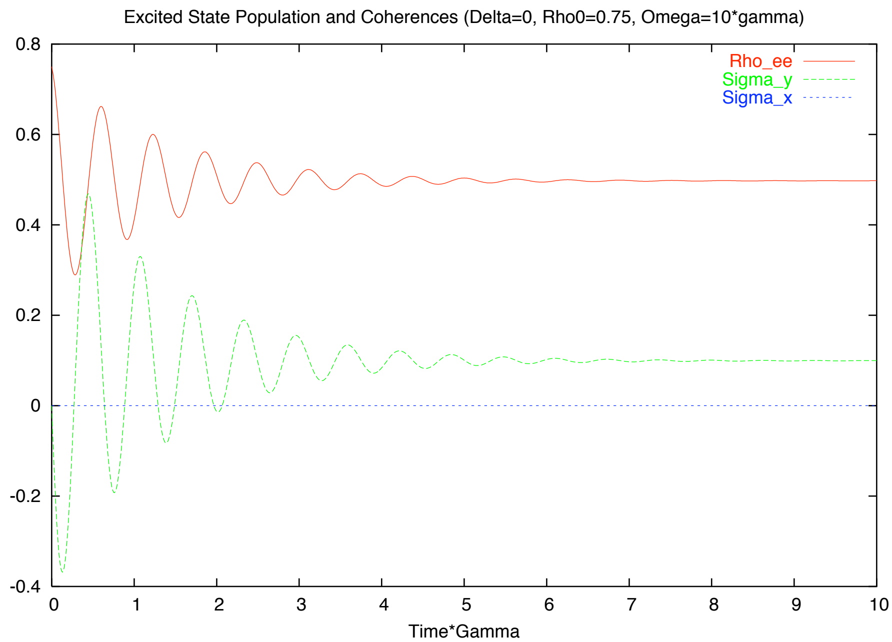

Figure 1:

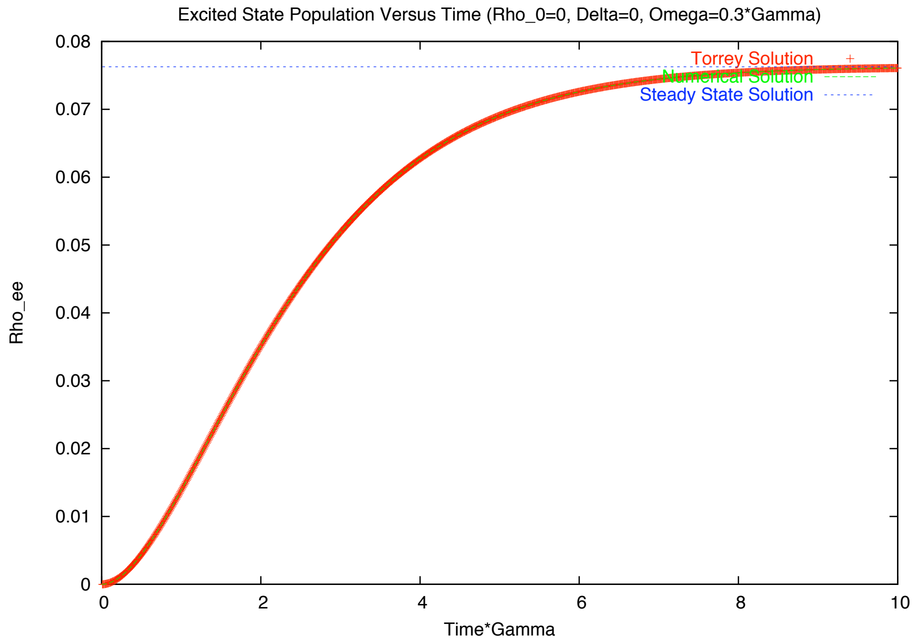

Figure 2:

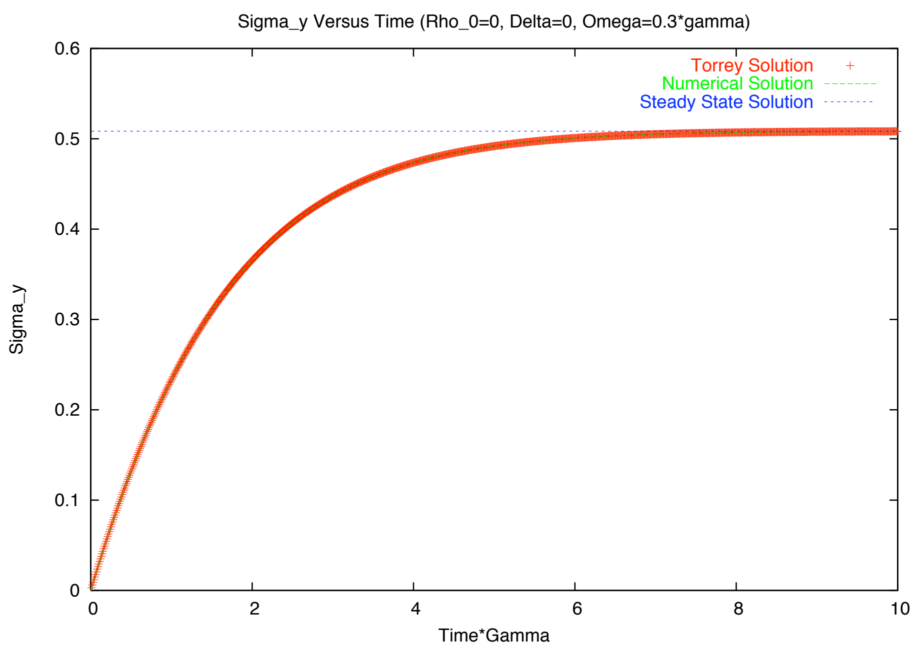

Figure 3:

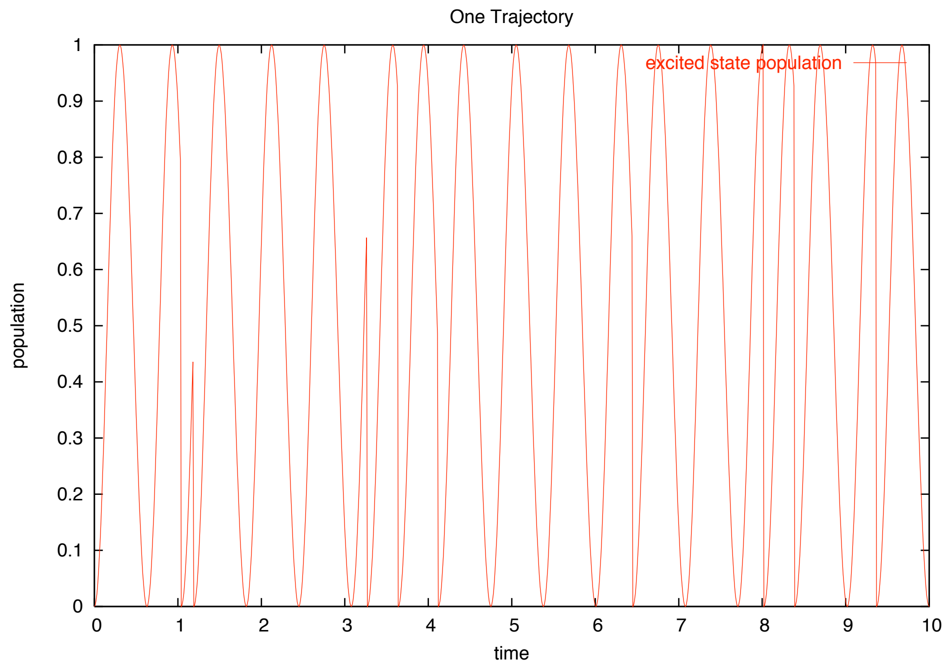

Figure 4:

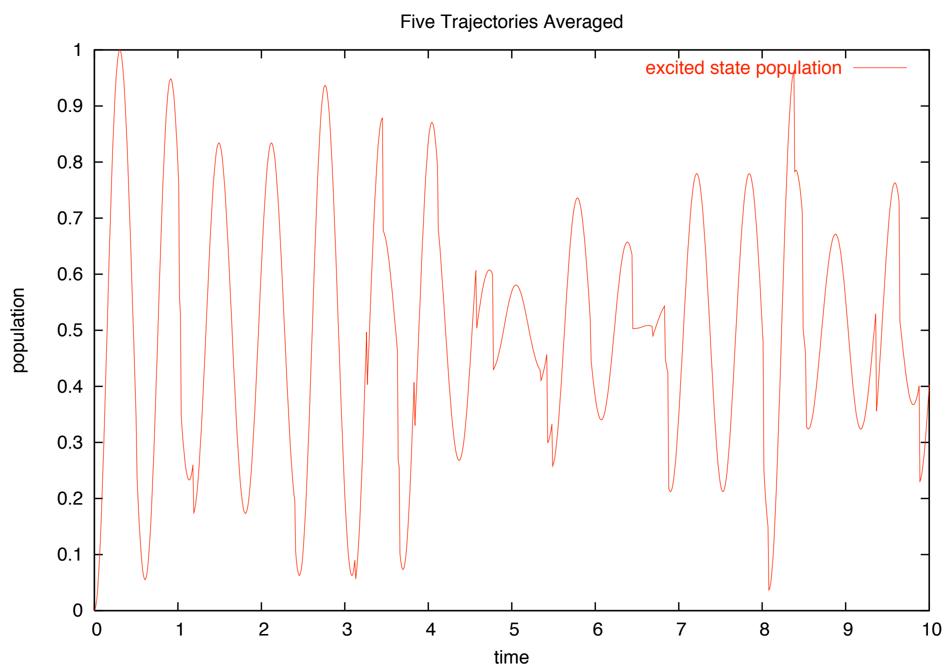

Figure 5:

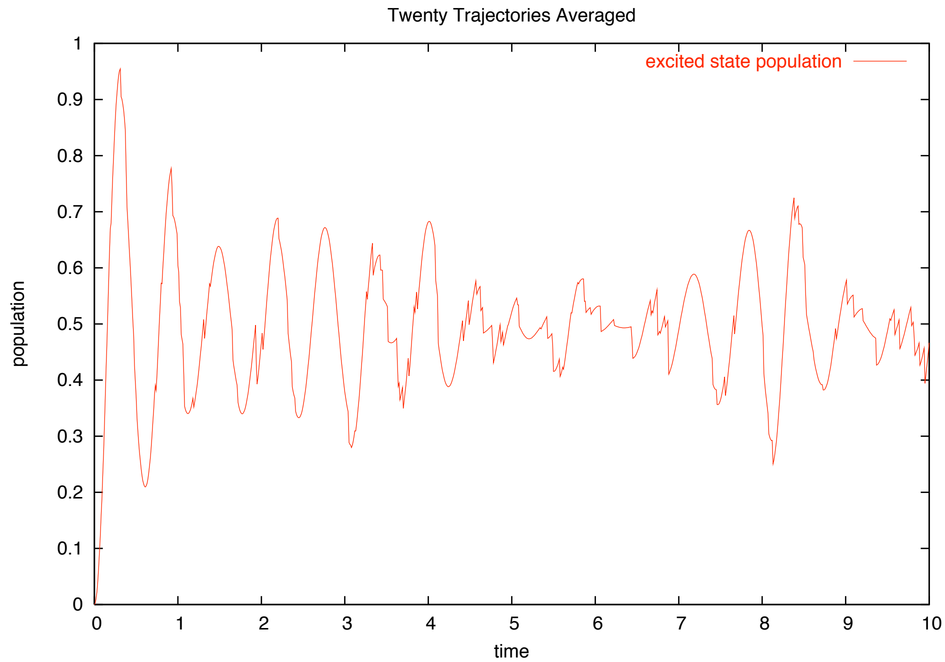

Figure 6:

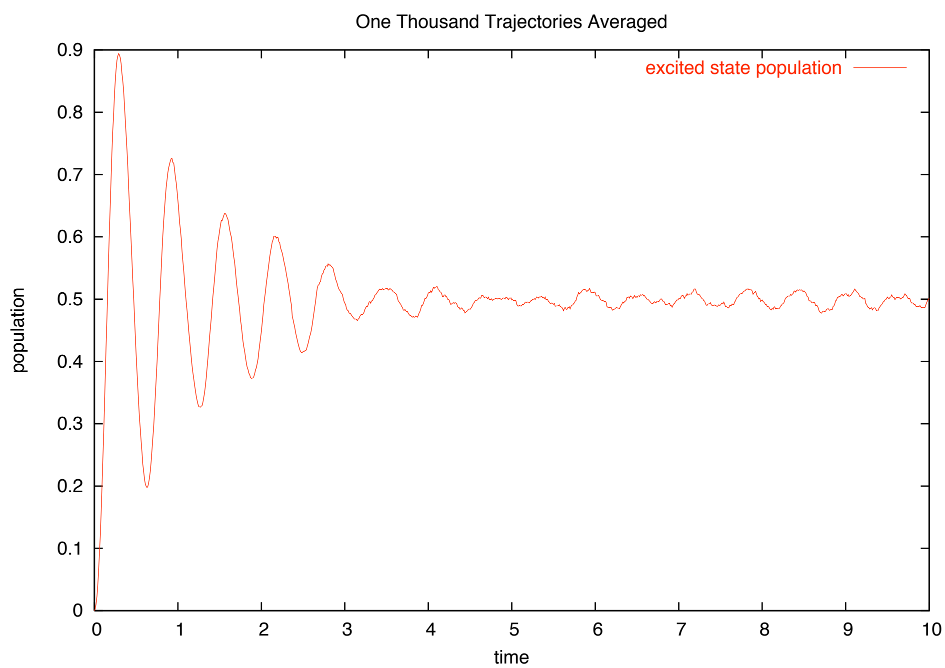

Figure 7:

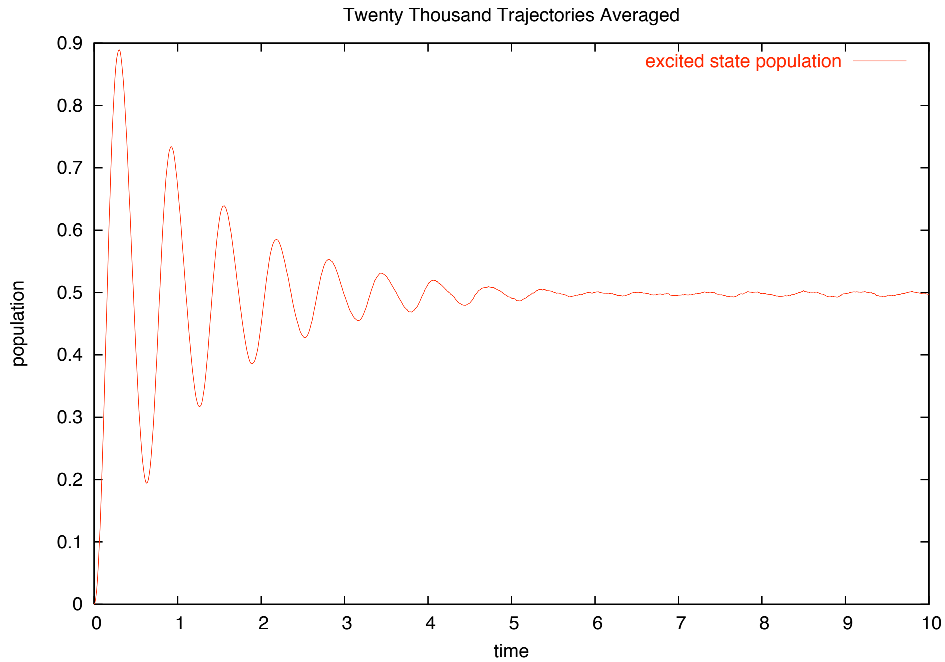

Figure 8:

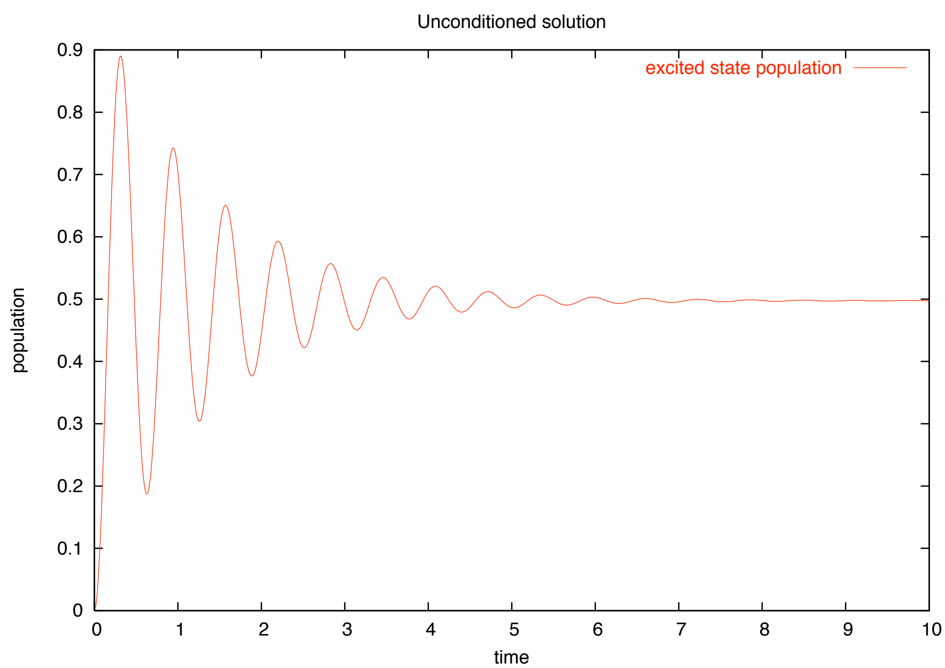

Figure 9:

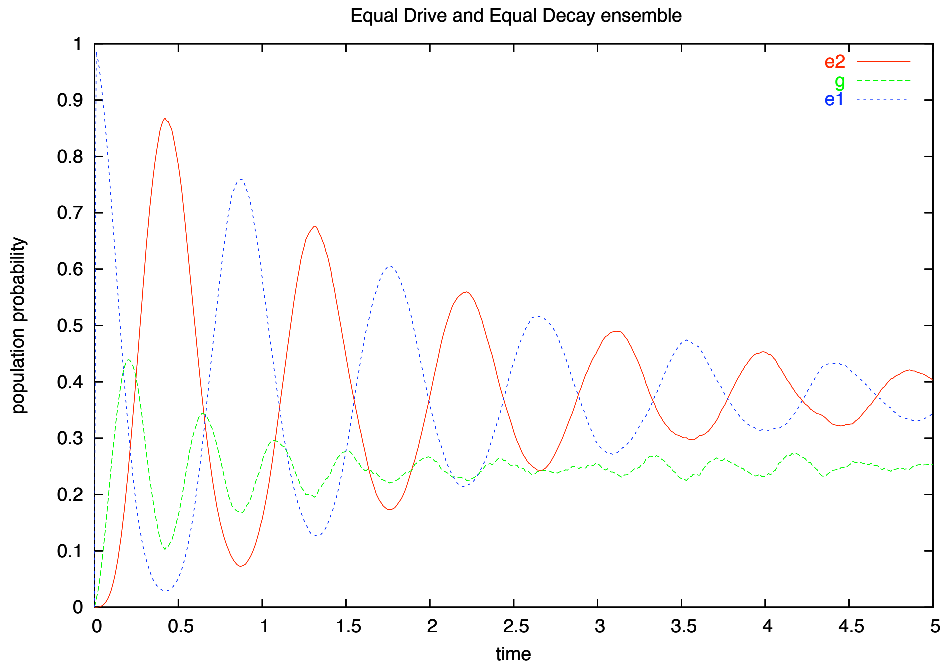

Figure 10:

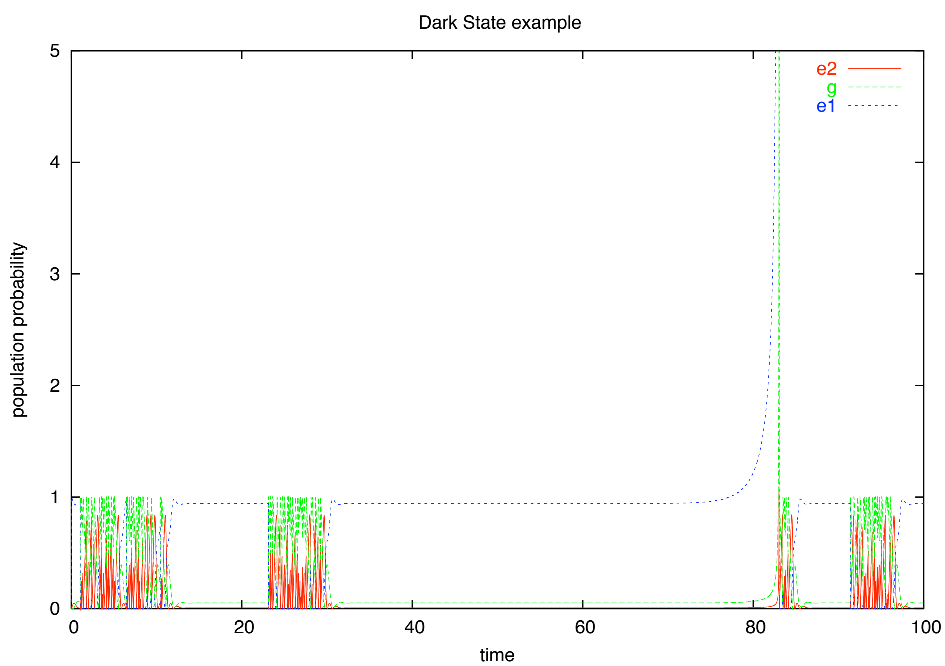

Figure 11:

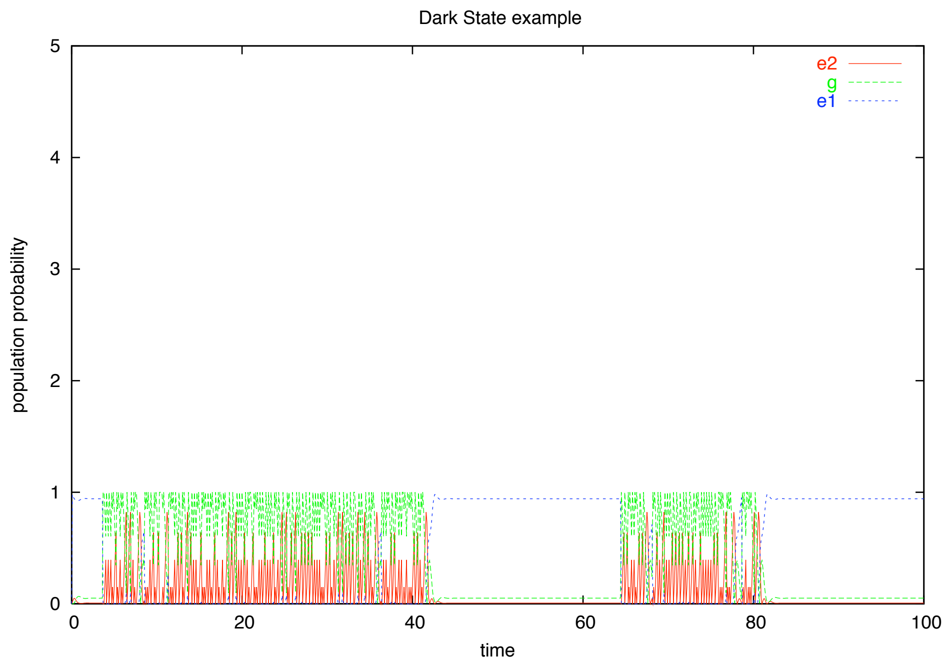
Dark State example
e2
g
e1
population probability
time
0
1
2
3
4
5
0
20
40
60
80
100

Figure 12: