Numerical Computation for Coherent Spin Flips In InAs Quantum Dots

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Abstract

We attempt to show that precise control of the Bloch vector is possible through numerical methods in a four level quantum system. We begin by setting the Bloch vector initially oriented along either its x, y, or z components and rotate the vector to another orientation. Arbitrary initial conditions are then chosen as well as arbitrary final rotated states to show that it may be possible to manipulate the Bloch vector for whatever conditions are demanded. Reasonable decay of both population and coherence along with broadening of energy levels due to collision is included. We show by numerical methods that relatively high fidelities can be reached.

1 Introduction

Several different systems have been proposed for using electrons as information carriers in quantum computing. One of the most prominent systems is a three level atom in λ formation, where the two ground states are non-degenerate and whose electron states are able to represent information through control of the populations and coherences of the ground states. The excited level is used to bridge the two ground states since any direct transition between the two ground states is forbidden. InAs quantum dots exhibit such an energy structure, yet with one complication. A fourth energy level exists very close to the excited level which produces some error in transitioning an electron between ground states. In other words, when one wants to transition an electron from one ground state to the other, there is a chance that this fourth state will decrease the probability of success. The goal then is to find a way to apply electric fields in such a way that the electrons can be manipulated as if they were in a three level system. A paper by Emary and Sham[1] is the primary source for this information and what follows. In addition to the formalism discussed in the Emary and Sham paper, population and coherence decay terms are added to the system as well as inhomogeneous broadening due to collisions of the electrons.

2 System

Consider a singly-charged self-assembled InAs quantum dot. When a magnetic field is applied in the x-direction, the two ground states Zeeman-split into $|x\pm\rangle$ with energy difference $E^e_{\mathrm{B}} = g^e_x \mu_{\mathrm{B}} B_x$. Heavy-hole component of the trion also splits into $|\tau\pm\rangle$ under this field and has the splitting $E^h_{\mathrm{B}} = -g^h_x \mu_{\mathrm{B}} B_x$, due to pseudo-spin. Recent measurements have given the magnitudes of these g-factors as $|g^e_x| = 0.46$ and $|g^h_x| = 0.29$[2].

Certain transitions are forbidden in this system, as seen in Fig. 1, and the polarizations of the allowed transitions are specified, which further simplifies the model. Similarly we can define the polarization vectors in terms of circular polarizations $\sigma_\pm$ as $V = (\sigma_- + \sigma_+)/\sqrt{2}$ and $H = (\sigma_- - \sigma_+)/\sqrt{2}$[3].To illuminate the

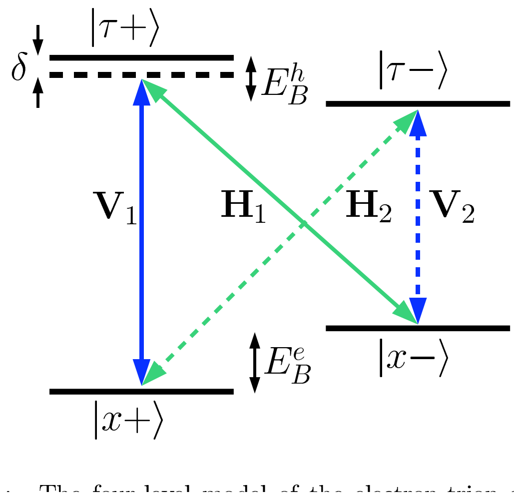

Figure 1: Four level energy structure. V and H are polarizations which only excite momentum conserving transitions.

system we use one V-polarized pulse with frequency ω_V and time-dependent Rabi frequency $\Omega_V(t)$, and one H-polarized pulse with frequency ω_H and Rabi frequency $\Omega_H(t)$, phase-locked and propagating in z-direction. Each pulse corresponds to two transitions, labelled 1 and 2 in Fig. 1. Therefore in the basis $\{|x+\rangle, |x-\rangle, |\tau+\rangle, |\tau-\rangle\}$ the Hamiltonian of this four-level system is:

$$H = \begin{pmatrix} +E_B^e/2 & 0 & \Omega_V^* e^{i\omega_V t + i\alpha} & \Omega_H^* e^{i\omega_H t} \\ 0 & -E_B^e/2 & \Omega_H^* e^{i\omega_H t} & \Omega_V^* e^{i\omega_V t + i\alpha} \\ \Omega_V e^{-i\omega_V t - i\alpha} & \Omega_H e^{-i\omega_H t} & E_\tau + E_B^h/2 & 0 \\ \Omega_H e^{-i\omega_H t} & \Omega_V e^{-i\omega_V t - i\alpha} & 0 & E_\tau - E_B^h/2 \end{pmatrix}, \tag{1}$$

where E_τ is the trion energy, α is the relative phase of the two lasers and $\hbar$ is set to be 1. By introducing

$$\begin{aligned} \Sigma_B &= (g_x^e + g_x^h)\mu_B B_x = E_B^e - E_B^h, \\ \Delta_B &= (g_x^e - g_x^h)\mu_B B_x = E_B^e + E_B^h, \end{aligned} \tag{2}$$

we write the frequencies of the two lasers as

$$\begin{aligned} \omega_V &= E_\tau - \Sigma_B/2 - \delta, \\ \omega_H &= E_\tau + \Delta_B/2 - \delta, \end{aligned} \tag{3}$$

where δ is the detuning, which is the same for each transition making the effective two level Raman transition on resonance. Using a rotating frame, the Hamiltonian becomes

$$H = \begin{pmatrix} 0 & 0 & \Omega_V^* e^{i\alpha} & \Omega_H^* e^{i\Delta_B t} \\ 0 & 0 & \Omega_H^* & \Omega_V^* e^{-i\Sigma_B t + i\alpha} \\ \Omega_V e^{-i\alpha} & \Omega_H & \delta & 0 \\ \Omega_H e^{-i\Delta_B t} & \Omega_V e^{i\Sigma_B t - i\alpha} & 0 & \delta \end{pmatrix}. \tag{4}$$

The parameters for InAs have been reported by Emary and Sham[1]. Under a magnetic field of $B = 1$T, the Zeeman-splitting of electron levels and heavy-hole trion levels are $E_B^e = -27$eV, $E_B^h = 17$eV. Population decay rate $\Gamma = 300$MHz and the pulse width is 100ps. These are the parameters used in the numerical calculations.

3 Formalism

The evolution of the system follows the master equation

$$d\rho = -\frac{i}{\hbar}[H, \rho] + D[\sqrt{\Gamma}\sigma]\rho dt. \tag{5}$$

If the decays are treated as indistinguishable, then we have

$$\sqrt{\Gamma}\sigma = \sqrt{\Gamma_{V1}}\sigma_{V1} + \sqrt{\Gamma_{V2}}\sigma_{V2} + \sqrt{\Gamma_{H1}}\sigma_{H1} + \sqrt{\Gamma_{H2}}\sigma_{H2}, \tag{6}$$

where

$$\begin{aligned} \sigma_{V1} &= |x+\rangle\langle\tau+|, \\ \sigma_{V2} &= |x-\rangle\langle\tau-|, \\ \sigma_{H1} &= |x-\rangle\langle\tau+|, \\ \sigma_{H2} &= |x+\rangle\langle\tau-|. \end{aligned} \tag{7}$$

For convenience we will take all of the four decay rates to be equal and scale everything by that quantity.

4 Bloch Vector

The Bloch vector is used to describe the populations and coherences of a two level system. In our four level system, the Bloch vector is used to describe the populations of the two ground states and their coherences with respect to each other. In the bases used for the above Hamiltonian, the components of the Bloch vector are related to the density matrix in the following way:

$$\begin{aligned} \langle\sigma_x\rangle &= 2Re[\rho_{12}], \\ \langle\sigma_y\rangle &= -2Im[\rho_{12}], \\ \langle\sigma_z\rangle &= \rho_{11} - \rho_{22}. \end{aligned} \tag{8}$$

When we talk about the x, y, and z components of the system we will be referring to these components of the Bloch vector.

The target of spin flip is to be able to flip any arbitrary state to a desired state; To start with we have chosen the three most simple cases, namely $+x$, $+y$ or $+z$ orientations as the initial conditions for the Bloch sphere.

5 Fidelity

A quantity called the fidelity is used to describe the closeness between the target state and the state we obtain by numerical integration of the master equation. It is defined as[4]

$$F = \overline{\langle\Psi_{\rm in}|\tilde{U}^\dagger(t)\rho_{\rm out}\tilde{U}(t)|\Psi_{\rm in}\rangle}, \tag{9}$$

where the overline represents an average over all input states $|\Psi_{\rm in}\rangle$,$\tilde{U}(t)$ is the time-evolution operator and $\rho_{\rm out}$ is the density matrix obtained by numerical integration. The fidelity is the value used to tell the genetic algorithm when the parameters it has chosen are desirable.

6 Algorithm

The process for finding the set of parameters utilized to orient the spin to a desired position was a genetic algorithm. The parameters to find were the two Rabi frequencies for the two ground states coupled to the same excited state, the phase between the two pulses and the common detuning from the excited state which is required for stimulated Raman transitions. In the language of the genetic algorithm we would say that there are 4 genes that we search for to manipulate the spin of our qubit. The initialization of the genes are somewhat random. There is a range in which the desired values may live, therefore, the algorithm randomly selects forty or more sets of genes to evaluate. The evaluation is determined by a ranking of the performance of the gene code. We want very little population to enter the excited state and a high fidelity in the outcome. These two parameters carry differing weights. For example, a perfect flip of the spins and a large amount of population transfered to the excited state would get a lower ranking than a perfect flip with little population transfer. The program was designed to evaluate a population of 40 and keep the top ten for breeding.

The next very important step is generating offspring from the best of the previous generation. This can be done numerous ways, often this process includes gene swapping between parents and a small amount of mutation. The program would make two children for every set of parents with randomly swapped genes and a small amount of mutation. Say parent 1 has genes a, b, c and d. While parent 2 has genes e, f, g, and h. The two offspring for this set of parents could look something like a', f', g', d' and e', b', c', h' or some-other variation. Where the primed genes are a slight mutation of the parents original gene. One other mechanism for generating a new generation that we us is asexual reproduction where the combination of genes from two or more parents isn't used only the random mutation cause from the wonderful random number generator rand pl().

If one of the offspring out performs a parent, our test places the offspring in the ordered hierarchy and shifts the less prominent members of society downward. These top ten again become the parents of a new generation. This is a tedious process taking many generations to optimize parameters. Some of the plots shown are for as few as 8 generations and as many as 50. The ideal case would be to run the simulation for several hundered generations or until a ultimate fidelity is met.

7 Results

We started with the most simple cases. These cases were starting the Bloch vector initially in the $+x$ direction, $+y$ direction, or $+z$ direction and rotating it to the $-x$, $-y$, or $-z$ direction respectively. (Figs. 2, 3, and 4) show some of these cases. The desired outcomes are produced with decent accuracy. It appears that rotating about the z-axis is more reliable for a desired final outcome than rotating about the x or y axes. It is important to note that the excited state populations are low for all times to reduce the effects of spontaneous emission since spontaneous emission randomizes coherences.

The next test was to start the Bloch vector in an arbitrary orientation and rotate it to some other arbitrary position. The initial position chosen was $(x, y, z) = (0.25, 0.31, 0.2)$. From here, rotations to three different final states were attempted. (Figs. 5, 6, and 7) depict the results for some cases.

The results are reasonable with a fidelity of approximately 90 percent. A better fidelity appears to be not possible to obtain. This is probably because the Bloch vector is not initially of unit magnitude. The populations in the ground states add to one initially, but the coherences are not of a magnitude such that the Bloch vector is normalized. Thus, by trying to get to the three chosen final states, one must induce coherence between the ground states, which is not possible without re-establishing coherence by driving population into the excited state. However, 90 percent fidelity for such a situation seems reasonably successful. Some results without requiring small excited state populations follow in the next section.

8 Additional on-Resonance Results

We initially align the Bloch vector parallel to $+x$, $+y$ and $+z$ directions in three individual cases. In these figures the excited state population is not required to be small. The results follow in (Fig. 8).

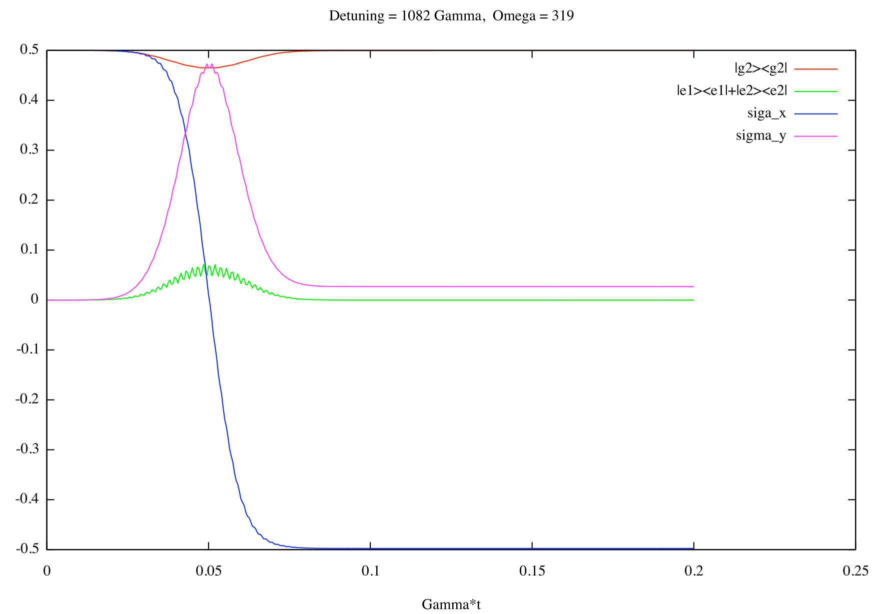

Figure 2: Bloch vector initially in $+x$ direction. Desire is to flip it from $(x, y, z) = (0.5, 0, 0.5)$ to $(-0.5, 0, 0.5)$.

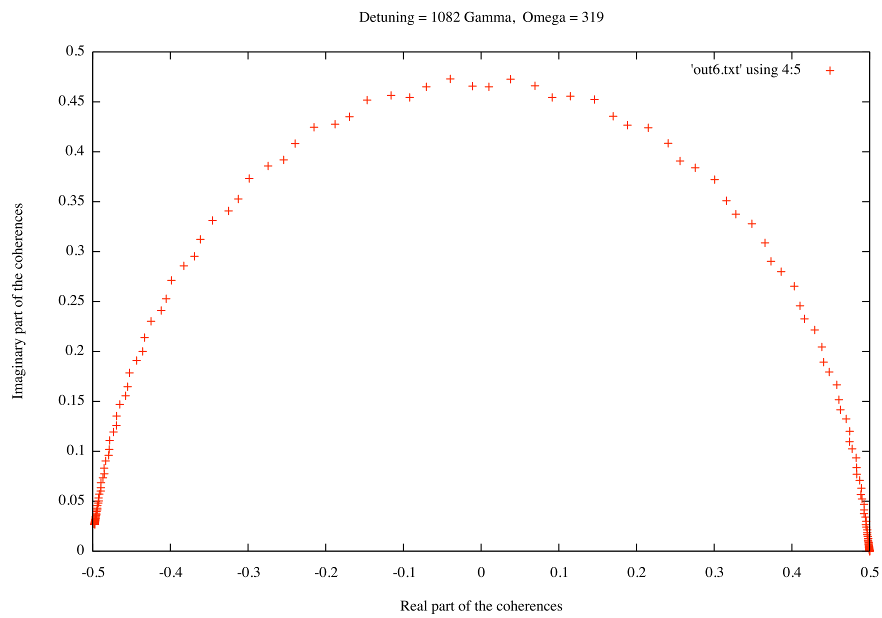

Figure 3: Plot of y versus x for the $+x$ to $-x$ flip. This graph shows that the transition is a rotation about the z-axis. The magnitude of the Bloch vector in the x,y plain is conserved.

References

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[3] C. Emary, X. Xu, D. G. Steel, S. Saikin, L. J. Sham, *Phys. Rev. Lett.* **98**, 047401 (2007).

[4] J. F. Poyatos, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **78**, 390(1997).

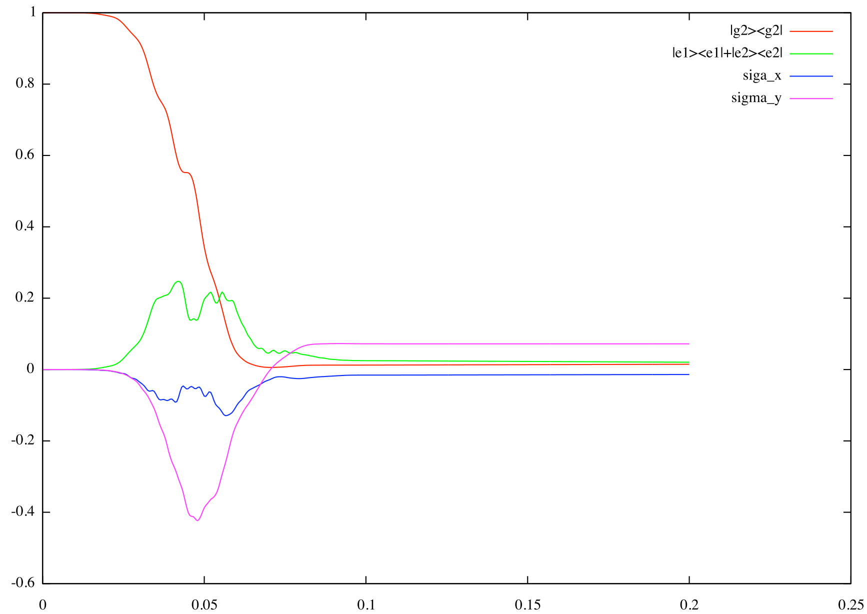


Figure 4: Bloch vector initially in $+z$ direction. Desire is to flip it from $(x, y, z) = (0, 0, 1)$ to $(0, 0, -1)$. The initial ground state population, excited state population, and coherences all go to about zero, leaving most of the population in the other ground state (not depicted, but evident from population conservation).

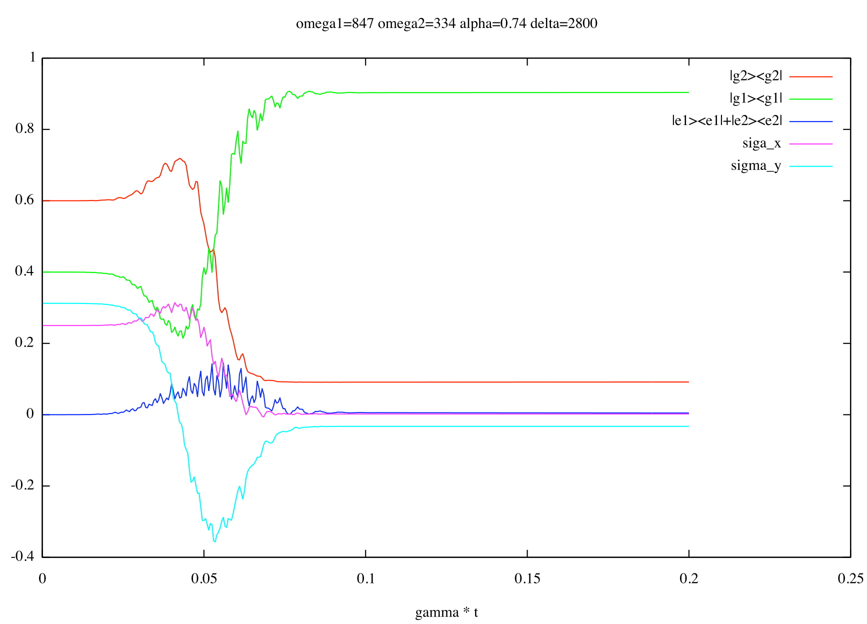


Figure 5: Rotation from $(x, y, z) = (0.25, 0.31, 0.2)$ to $(0, 0, -1)$

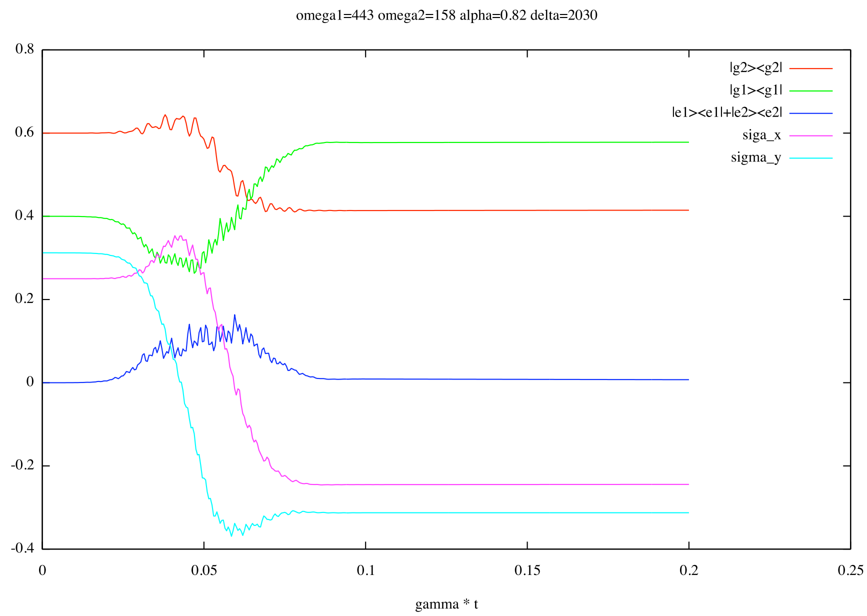


Figure 6: Rotation from $(x, y, z) = (0.25, 0.31, 0.2)$ to $(-0.25, -0.31, -0.2)$

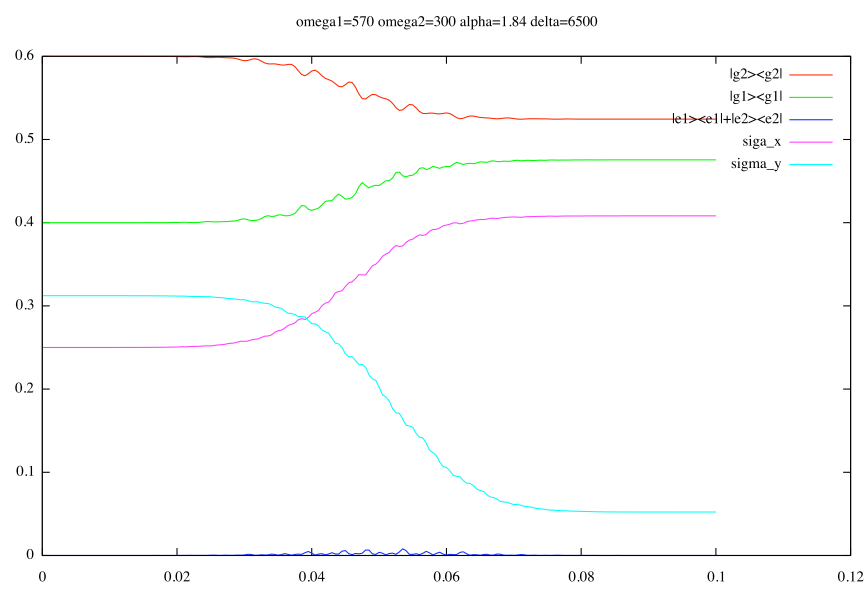
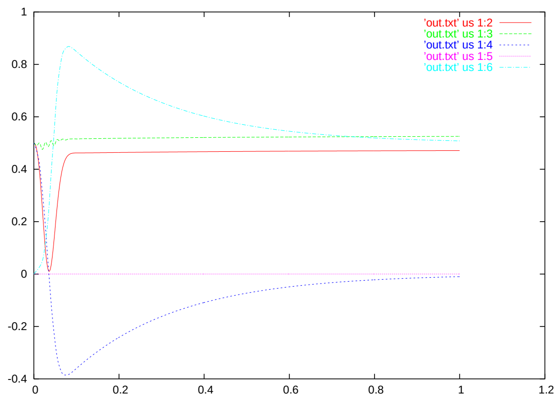
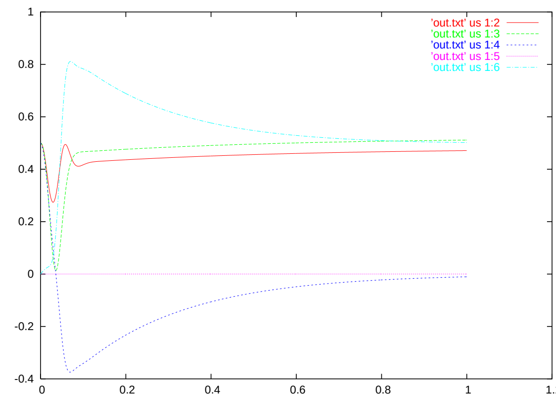


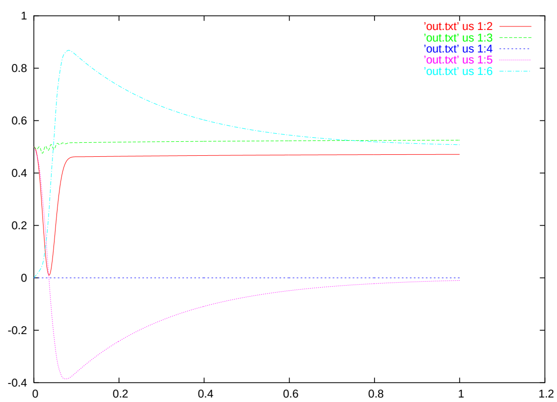
Figure 7: Rotation from $(x, y, z) = (0.25, 0.31, 0.2)$ to $(1, 0, 0)$



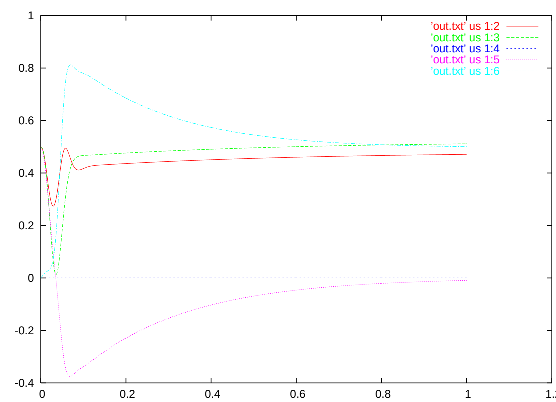
(a) x flip with V pulse



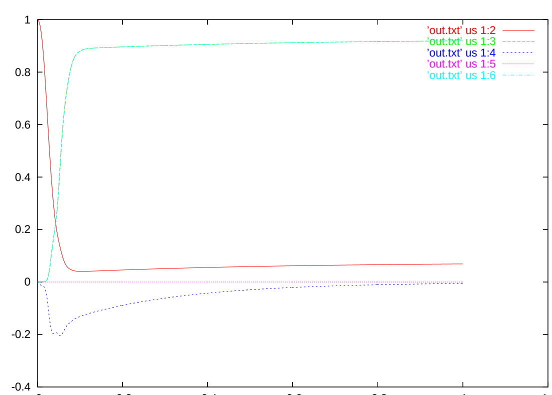
(b) x flip with H pulse



(c) y flip with V pulse



(d) y flip with H pulse



(e) z flip with V and H pulses

Figure 8: Columns 2 to 6 correspond to population in $|x+\rangle$ state, population in $|x-\rangle$ state, real and imaginary part of the coherence between $|x+\rangle$ and $|x-\rangle$ states, and fidelity. In (a) and (b), the coherence is flipped by π using V and H pulse respectively. So in a Bloch sphere constructed by the two ground states, the spin initially points to $+x$ direction and ends up pointing to $-x$ direction. (c) and (d) are same as (a) and (b) except that the spin goes from $+y$ to $-y$ direction. In (e), however, a H pulse is applied immediately after a V pulse, which pump the population back and forth between one excited state and two ground states.