Two-Level Systems (short version)

- qubit, $|0\rangle$, $|1\rangle$ (two-level atom) (pHe in double-well potential)
- general Hamiltonian? Hermitian $\begin{pmatrix} E_1 & \alpha \\ \alpha^* & E_0 \end{pmatrix}$ $E_{0,1} \in \mathbb{R}$

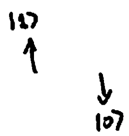
  $E_{0,1} \to$ (uncoupled) energies of $|0,1\rangle$

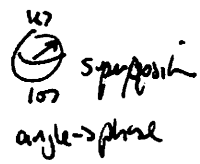
  $\alpha \to$ coupling rate / transition rate

  e.g. $\alpha = -\frac{\vec{d}\cdot\vec{E}}{2}$ for TLA + laser field.

- Bloch sphere: represent $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$ (pure states) on (Feynman-Vernon-Hellwarth representation) unit sphere

define: $\vec{\Psi} := u\hat{x} + v\hat{y} + w\hat{z}$





$$u := \rho_{10} + \rho_{01} = c_1 c_0^* + \text{c.c.} \quad \to$$
$$v := i(\rho_{10} - \rho_{01}) = i(c_1 c_0^* - \text{c.c.}) \quad \to \Big\} \text{(coherences) (quadratures)}$$
$$w := \rho_{11} - \rho_{00} = |c_1|^2 - |c_0|^2 \quad \to \text{population}$$

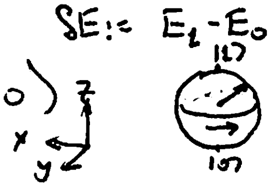
$$|\vec{\Psi}|^2 = u^2 + v^2 + w^2 = (c_1 c_0^* + c_0 c_1^*)^2 - (c_1 c_0^* - c_0 c_1^*)^2 + (|c_1|^2 - |c_0|^2)^2$$
$$= 4 c_1 c_0^* c_0 c_1^* + |c_1|^4 + |c_0|^4 - 2|c_1|^2|c_0|^2$$
$$= (|c_1|^2 + |c_0|^2)^2 = 1. \quad \Rightarrow \text{unit sphere}$$

(can show: any unitary transformation on $\vec{\Psi}$ is a rotation of sphere.
(state is point on sphere)

mathematically: $\dot{\vec{\Psi}} = \vec{\Omega} \times \vec{\Psi}$ (like precession: $\vec{\tau} = \dot{\vec{L}} = \vec{\Omega} \times \vec{L}$)

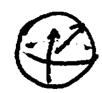
$$\vec{\Omega} := 2|\alpha|\hat{x} + \frac{\delta E}{\hbar}\hat{z}$$
$$\delta E := E_1 - E_0$$

Phase evolution ($\delta E \neq 0$, $\alpha = 0$)



$e^{-iE_1t/\hbar}$, $e^{-iE_0t/\hbar}$

Resonant coupling: ($\delta E = 0$, $\alpha \neq 0$)



"Rabi flopping" $|0\rangle \to |1\rangle \to |0\rangle$

(Combined) Off-resonant coupling ($\delta E \neq 0$, $\alpha \neq 0$)

incomplete Rabi flopping.